Chapter 3

# Systems of Equations and Inequalities

What You'll Learn

- **Lessons 3-1, 3-2, and 3-5** Solve systems of linear equations in two or three variables.
- **Lesson 3-3** Solve systems of inequalities.
- **Lesson 3-4** Use linear programming to find maximum and minimum values of functions.

Why It's Important

Systems of linear equations and inequalities can be used to model real-world situations in which many conditions must be met. For example, hurricanes are classified using inequalities that involve wind speed and storm surge. Weather satellites provide images of hurricanes, which are rated on a scale of 1 to 5. *You will learn how to classify the strength of a hurricane in Lesson 3-3.*

Key Vocabulary

- **system of equations** (p. 110)
- **substitution method** (p. 116)
- **elimination method** (p. 118)
- **linear programming** (p. 130)
- **ordered triple** (p. 136)

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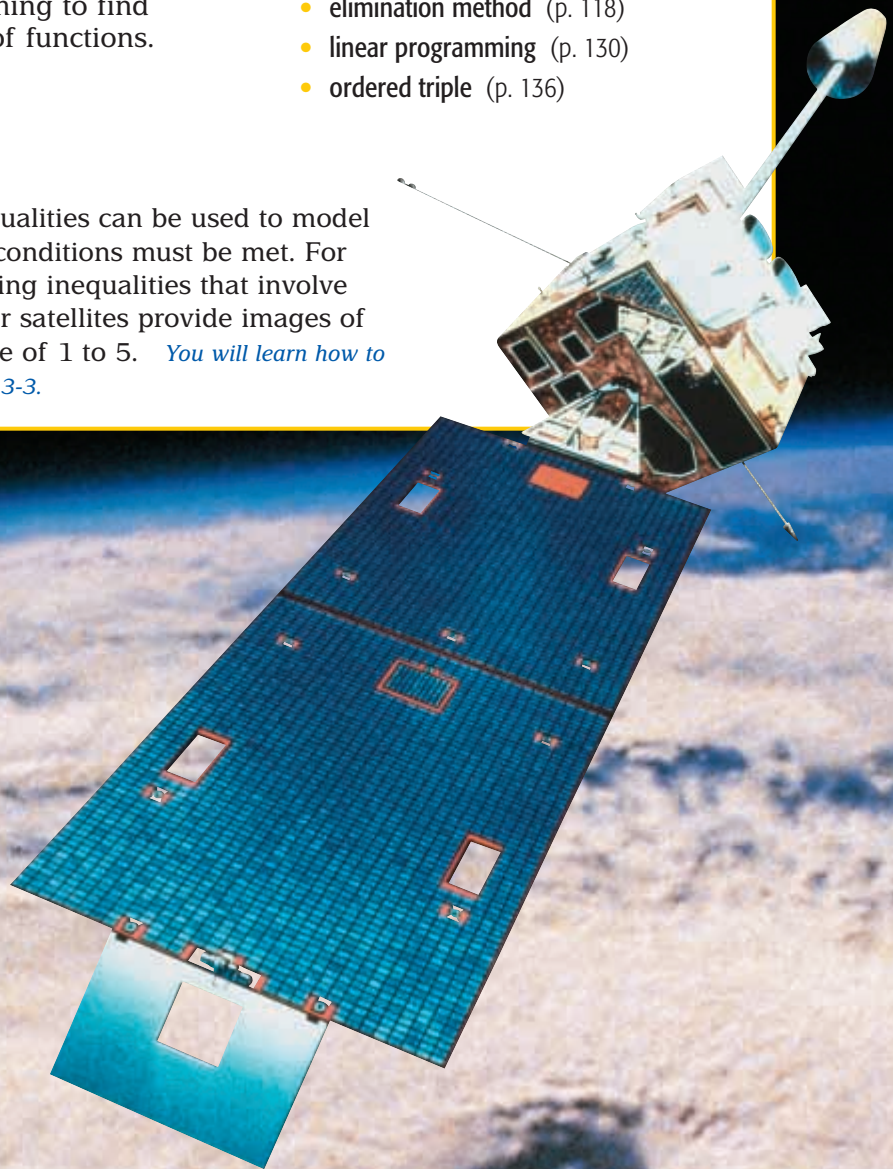
Getting Started

▶ **Prerequisite Skills** To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 3.

For Lesson 3-1   Graph Linear Equations

Graph each equation.   *(For review, see Lesson 2-2.)*

1. $2y = x$ **2.** $y = x - 4$ **3.** $y = 2x - 3$

4. $x + 3y = 6$ **5.** $2x + 3y = -12$ **6.** $4y - 5x = 10$

For Lesson 3-2   Solve for a Specified Variable

Solve each equation for y.   *(For review, see Lesson 1-3.)*

7. $2x + y = 0$ **8.** $x - y = -4$ **9.** $6x + 2y = 12$

10. $8 - 4y = 5x$ **11.** $\frac{1}{2}y + 3x = 1$ **12.** $\frac{1}{3}x - 2y = 8$

For Lessons 3-3 and 3-4   Graph Inequalities

Graph each inequality.   *(For review, see Lesson 2-7.)*

13. $y \geq -2$ **14.** $x + y \leq 0$ **15.** $y < 2x - 2$

16. $x + 4y < 3$ **17.** $2x - y \geq 6$ **18.** $3x - 4y < 10$

For Lesson 3-5   Evaluate Expressions

Evaluate each expression if $x = -3$, $y = 1$, and $z = 2$.   *(For review, see Lesson 1-1.)*

19. $3x + 2y - z$ **20.** $3y - 8z$ **21.** $x - 5y + 4z$

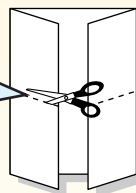**22.** $2x + 9y + 4z$ **23.** $2x - 6y - 5z$ **24.** $7x - 3y + 2z$

FOLDABLES™
Study Organizer

Make this Foldable to record information about systems of linear equations and inequalities. Begin with one sheet of 11" × 17" paper and four sheets of grid paper.
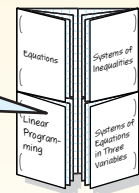
Step 1   **Fold and Cut**

Fold the short sides of the 11" × 17" paper to meet in the middle. Cut each tab in half as shown.

Step 2   **Staple and Label**

Insert 2 folded half-sheets of grid paper in each tab. Staple at edges. Label each tab as shown.

Reading and Writing   As you read and study the chapter, fill the tabs with notes, diagrams, and examples for each topic.

3-1 Solving Systems of Equations by Graphing

How can a system of equations be used to predict sales?

Since 1999, the growth of in-store sales for Custom Creations by Cathy can be modeled by $y = 4.2x + 29$, and the growth of her online sales can be modeled by $y = 7.5x + 9.3$, where x represents the number of years since 1999 and y represents the amount of sales in thousands of dollars. The equations $y = 4.2x + 29$ and $y = 7.5x + 9.3$ are called a system of equations.
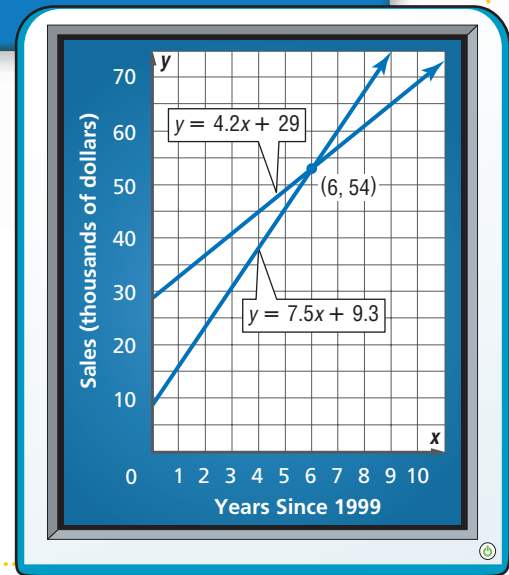
GRAPH SYSTEMS OF EQUATIONS A **system of equations** is two or more equations with the same variables. To solve a system of equations, find the ordered pair that satisfies all of the equations. One way to do this is to graph the equations on the same coordinate plane. The point of intersection represents the solution.

Example 1 Solve by Graphing

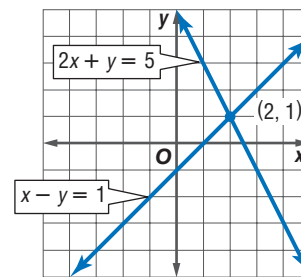Solve the system of equations by graphing.
$$2x + y = 5$$
$$x - y = 1$$

Write each equation in slope-intercept form.

$$2x + y = 5 \rightarrow y = -2x + 5$$
$$x - y = 1 \rightarrow y = x - 1$$

The graphs appear to intersect at (2, 1).

CHECK Substitute the coordinates into each equation.

$2x + y = 5$	$x - y = 1$	Original equations
$2(2) + 1 \stackrel{?}{=} 5$	$2 - 1 \stackrel{?}{=} 1$	Replace x with 2 and y with 1.
$5 = 5$ ✓	$1 = 1$ ✓	Simplify.

The solution of the system is (2, 1).

Systems of equations are used in businesses to determine the *break-even point*. The break-even point is the point at which the income equals the cost. If a business is operating at the break-even point, it is neither making nor losing money.

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Example 2 Break-Even Point Analysis

MUSIC Travis and his band are planning to record their first CD. The initial start-up cost is $1500, and each CD will cost $4 to produce. They plan to sell their CDs for $10 each. How many CDs must the band sell before they make a profit?

Let x = the number of CDs, and let y = the number of dollars.

Cost of x CDs	is	cost per CD	plus	startup cost.
y	$=$	$4x$	$+$	1500

Income from x CDs	is	price per CD	times	number of CDs.
y	$=$	10	$\cdot$	x
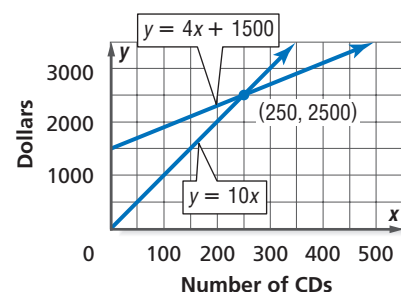
The graphs intersect at (250, 2500). This is the break-even point. If the band sells fewer than 250 CDs, they will lose money. If the band sells more than 250 CDs, they will make a profit.

CLASSIFY SYSTEMS OF EQUATIONS Graphs of systems of linear equations may be intersecting lines, parallel lines, or the same line. A system of equations is **consistent** if it has at least one solution and **inconsistent** if it has no solutions. A consistent system is **independent** if it has exactly one solution or **dependent** if it has an infinite number of solutions.

Example 3 Intersecting Lines

Graph the system of equations and describe it as *consistent and independent, consistent and dependent,* or *inconsistent.*
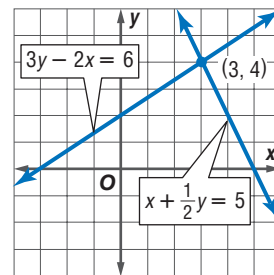
$$x + \frac{1}{2}y = 5$$
$$3y - 2x = 6$$

Write each equation in slope-intercept form.

$$x + \frac{1}{2}y = 5 \rightarrow y = -2x + 10$$

$$3y - 2x = 6 \rightarrow y = \frac{2}{3}x + 2$$

The graphs intersect at (3, 4). Since there is one solution, this system is *consistent and independent*.

Example 4 Same Line

Graph the system of equations and describe it as *consistent and independent, consistent and dependent,* or *inconsistent.*
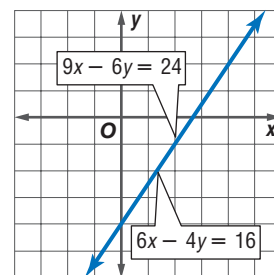
$$9x - 6y = 24$$
$$6x - 4y = 16$$

$$9x - 6y = 24 \rightarrow y = \frac{3}{2}x - 4$$

$$6x - 4y = 16 \rightarrow y = \frac{3}{2}x - 4$$

Since the equations are equivalent, their graphs are the same line. Any ordered pair representing a point on that line will satisfy both equations.

So, there are infinitely many solutions to this system. This system is *consistent and dependent*.

Example 5 Parallel Lines

Graph the system of equations and describe it as *consistent and independent,* *consistent and dependent,* or *inconsistent.*
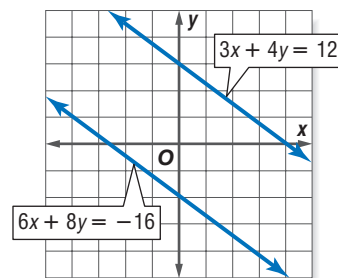
$3x + 4y = 12$
$6x + 8y = -16$

$3x + 4y = 12 \rightarrow y = -\dfrac{3}{4}x + 3$

$6x + 8y = -16 \rightarrow y = -\dfrac{3}{4}x - 2$

The lines do not intersect. Their graphs are parallel lines. So, there are no solutions that satisfy both equations. This system is *inconsistent.*
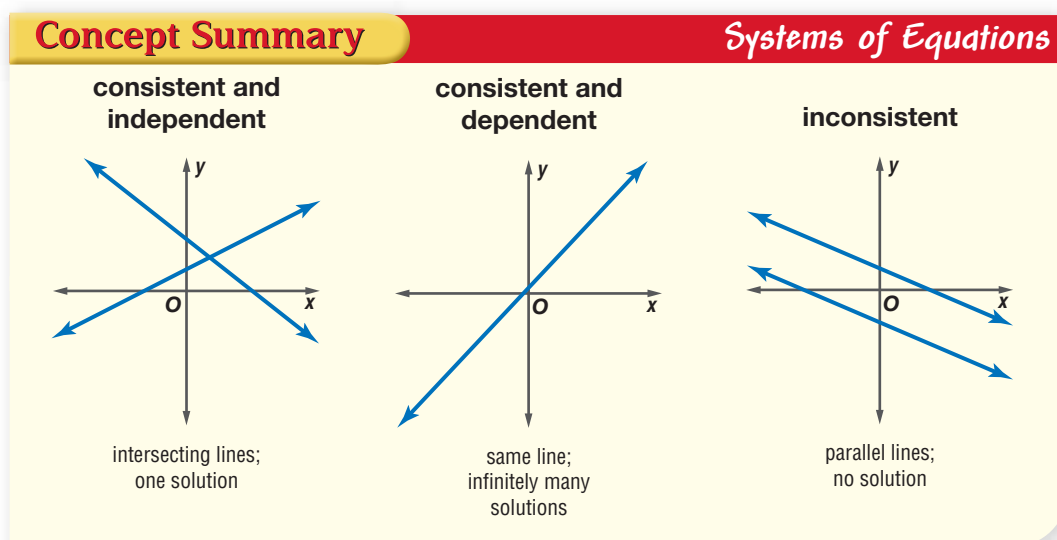
The relationship between the graph of a system of equations and the number of its solutions is summarized below.

Concept Summary   Systems of Equations

consistent and independent

intersecting lines;
one solution

consistent and dependent

same line;
infinitely many
solutions

inconsistent

parallel lines;
no solution

Check for Understanding

Concept Check

1. **Explain** why a system of linear equations cannot have exactly two solutions.
2. **OPEN ENDED** Give an example of a system of equations that is consistent and independent.
3. **Explain** why it is important to check a solution found by graphing in both of the original equations.

Guided Practice

Solve each system of equations by graphing.

4. $y = 2x + 9$
   $y = -x + 3$

5. $3x + 2y = 10$
   $2x + 3y = 10$

6. $4x - 2y = 22$
   $6x + 9y = -3$

Graph each system of equations and describe it as *consistent and independent,* *consistent and dependent,* or *inconsistent.*
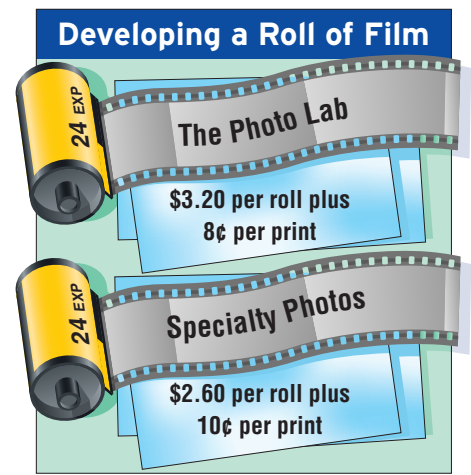
7. $y = 6 - x$
   $y = x + 4$

8. $x + 2y = 2$
   $2x + 4y = 8$

9. $x - 2y = 8$
   $\dfrac{1}{2}x - y = 4$

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PHOTOS For Exercises 10–12, use the graphic at the right.

Developing a Roll of Film

The Photo Lab
$3.20 per roll plus
8¢ per print

Specialty Photos
$2.60 per roll plus
10¢ per print

10. Write equations that represent the cost of developing a roll of film at each lab.

11. Under what conditions is the cost to develop a roll of film the same for either store?

12. When is it best to use The Photo Lab and when is it best to use Specialty Photos?

Practice and Apply

Solve each system of equations by graphing.

13. $y = 2x - 4$
$y = -3x + 1$

14. $y = 3x - 8$
$y = x - 8$

15. $x + 2y = 6$
$2x + y = 9$

16. $2x + 3y = 12$
$2x - y = 4$

17. $3x - 7y = -6$
$x + 2y = 11$

18. $5x - 11 = 4y$
$7x - 1 = 8y$

19. $2x + 3y = 7$
$2x - 3y = 7$

20. $8x - 3y = -3$
$4x - 2y = -4$

21. $\frac{1}{4}x + 2y = 5$
$2x - y = 6$

22. $\frac{2}{3}x + y = -3$

$y - \frac{1}{3}x = 6$

23. $\frac{1}{2}x - y = 0$

$\frac{1}{4}x + \frac{1}{2}y = -2$

24. $\frac{4}{3}x + \frac{1}{5}y = 3$

$\frac{2}{3}x - \frac{3}{5}y = 5$

Graph each system of equations and describe it as *consistent and independent,* *consistent and dependent,* **or** *inconsistent.*

25. $y = x + 4$
$y = x - 4$

26. $y = x + 3$
$y = 2x + 6$

27. $x + y = 4$
$-4x + y = 9$

28. $3x + y = 3$
$6x + 2y = 6$

29. $y - x = 5$
$2y - 2x = 8$

30. $4x - 2y = 6$
$6x - 3y = 9$

31. $2y = x$
$8y = 2x + 1$

32. $2y = 5 - x$
$6y = 7 - 3x$

33. $0.8x - 1.5y = -10$
$1.2x + 2.5y = 4$

34. $1.6y = 0.4x + 1$
$0.4y = 0.1x + 0.25$

35. $3y - x = -2$
$y - \frac{1}{3}x = 2$

36. $2y - 4x = 3$
$\frac{4}{3}x - y = -2$

37. **GEOMETRY** The sides of an angle are parts of two lines whose equations are $2y + 3x = -7$ and $3y - 2x = 9$. The angle's vertex is the point where the two sides meet. Find the coordinates of the vertex of the angle.

38. **GEOMETRY** The graphs of $y - 2x = 1$, $4x + y = 7$, and $2y - x = -4$ contain the sides of a triangle. Find the coordinates of the vertices of the triangle.

TRAVEL For Exercises 39–41, use the following information.
Adam and his family are planning to rent a midsize car for a one-day trip. In the Standard Rental Plan, they can rent a car for $52 per day plus 23 cents per mile. In the Deluxe Rental Plan, they can rent a car for $80 per day with unlimited mileage.
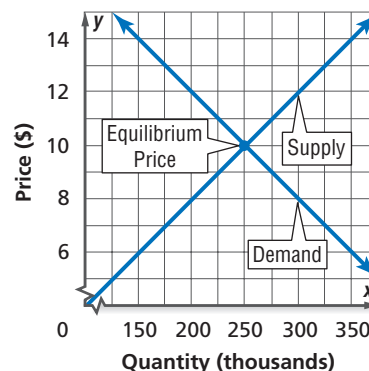
39. For each plan, write an equation that represents the cost of renting a car.

40. Graph the equations. Estimate the break-even point of the rental costs.

41. If Adam's family plans to drive 150 miles, which plan should they choose?

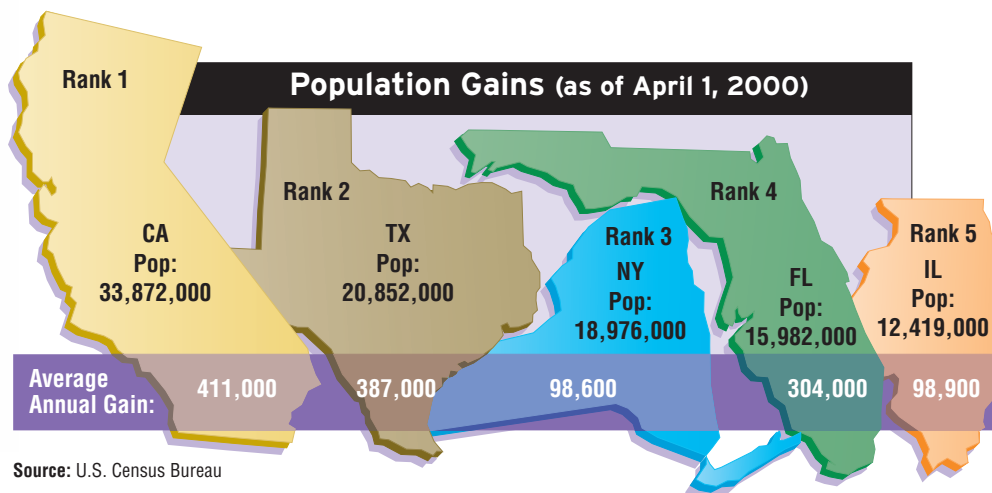www.algebra2.com/self_check_quiz

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ECONOMICS For Exercises 42–44, use the graph below that shows the supply and demand curves for a new multivitamin.

In Economics, the point at which the supply equals the demand is the *equilibrium price*. If the supply of a product is greater than the demand, there is a surplus and prices fall. If the supply is less than the demand, there is a shortage and prices rise.

42. If the price for vitamins is $8 a bottle, what is the supply of the product and what is the demand? Will prices tend to rise or fall?

43. If the price for vitamins is $12 a bottle, what is the supply of the product and what is the demand? Will prices tend to rise or fall?

44. At what quantity will the prices stabilize? What is the equilibrium price for this product?

Population

In the United States there is approximately one birth every 8 seconds and one death every 14 seconds.

Source: U.S. Census Bureau

POPULATION For Exercises 45–47, use the graphic that shows 2000 state populations.

Population Gains (as of April 1, 2000)

	Rank 1 CA Pop: 33,872,000	Rank 2 TX Pop: 20,852,000	Rank 3 NY Pop: 18,976,000	Rank 4 FL Pop: 15,982,000	Rank 5 IL Pop: 12,419,000
Average Annual Gain:	411,000	387,000	98,600	304,000	98,900

Source: U.S. Census Bureau

45. Write equations that represent the populations of Florida and New York x years after 2000. Assume that both states continue to gain the same number of residents every year. Let y equal the population in thousands.

46. Graph both equations for the years 2000 to 2020. Estimate when the populations of both states will be equal.

47. Do you think Florida will overtake New York as the third most populous state by 2010? by 2020? Explain your reasoning.

48. CRITICAL THINKING State the conditions for which the system below is: (a) consistent and dependent, (b) consistent and independent, (c) inconsistent.

$ax + by = c$
$dx + ey = f$

49. WRITING IN MATH   Answer the question that was posed at the beginning of the lesson.

How can a system of equations be used to predict sales?

Include the following in your answer:
- an explanation of the real-world meaning of the solution of the system of equations in the application at the beginning of the lesson, and
- a description of what a business owner would learn if the system of equations representing the in-store and online sales is inconsistent.

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50. What are the coordinates (x, y) at which the graphs of $2x + 3y = 12$ and $2x - y = 4$ intersect?

Ⓐ $(3, 2)$ Ⓑ $(2, 3)$ Ⓒ $(1, -2)$ Ⓓ $(-3, 6)$

51. Which equation has the same graph as $4x + 8y = 12$?

Ⓐ $x + y = 3$ Ⓑ $2x + y = 3$ Ⓒ $x + 2y = 3$ Ⓓ $2x + 2y = 6$

Graphing Calculator

INTERSECT FEATURE To use a TI-83 Plus to solve a system of equations, graph both equations on the same screen. Then, select intersect, which is option 5 under the CALC menu, to find the coordinates of the point of intersection. Solve each system of equations to the nearest hundredth.

52. $y = 0.125x - 3.005$
$y = -2.58$

53. $3.6x - 2y = 4$
$-2.7x + y = 3$

54. $y = 0.18x + 2.7$
$y = -0.42x + 5.1$

55. $1.6x + 3.2y = 8$
$1.2x + 2.4y = 4$

56. $y - \dfrac{1}{4}x = 6$
$2y + \dfrac{1}{2}x = 3$

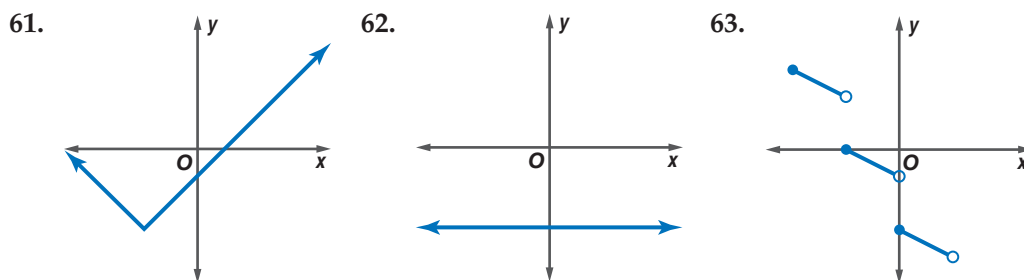**57.** $\dfrac{1}{2}y - 5x = 8$
$\dfrac{1}{3}y - 8x = -7$

Maintain Your Skills

Mixed Review

Graph each inequality. *(Lesson 2-7)*

58. $y \geq 5 + 3x$ **59.** $2x + y > -4$ **60.** $2y - 1 \leq x$

Identify each function as S for step, C for constant, A for absolute value, or P for piecewise. *(Lesson 2-6)*

61.

62.

63.

Solve each equation. Check your solutions. *(Lesson 1-4)*

64. $|x| - 5 = 8$ **65.** $|w + 3| = 12$ **66.** $|6a - 4| = -2$

67. $3|2t - 1| = 15$ **68.** $|4r + 3| - 7 = 10$ **69.** $|k + 7| = 3k - 11$

Write an algebraic expression to represent each verbal expression. *(Lesson 1-3)*

70. the sum of 8 and 2 times a number

71. six less than the square of a number

72. four times the sum of a number and 5

73. the quotient of a number and 3 increased by 1

Getting Ready for the Next Lesson

PREREQUISITE SKILL Simplify each expression.
*(To review **simplifying expressions**, see Lesson 1-2.)*

74. $(3x + 5) - (2x + 3)$ **75.** $(3y - 11) + (6y + 12)$

76. $(5x - y) + (-8x + 7y)$ **77.** $6(2x + 3y - 1)$

78. $5(4x + 2y - x + 2)$ **79.** $3(x + 4y) - 2(x + 4y)$

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3-2 Solving Systems of Equations Algebraically

What You'll Learn

- Solve systems of linear equations by using substitution.
- Solve systems of linear equations by using elimination.

Vocabulary
- substitution method
- elimination method

## How can systems of equations be used to make consumer decisions?

In January, Yolanda's long-distance bill was $5.50 for 25 minutes of calls. The bill was $6.54 in February, when Yolanda made 38 minutes of calls. What are the rate per minute and flat fee the company charges?

Let x equal the rate per minute, and let y equal the monthly fee.

January bill: $25x + y = 5.5$
February bill: $38x + y = 6.54$

It is difficult to determine the exact coordinates of the point where the lines intersect from the graph. For systems of equations like this one, it may be easier to solve the system by using algebraic methods.
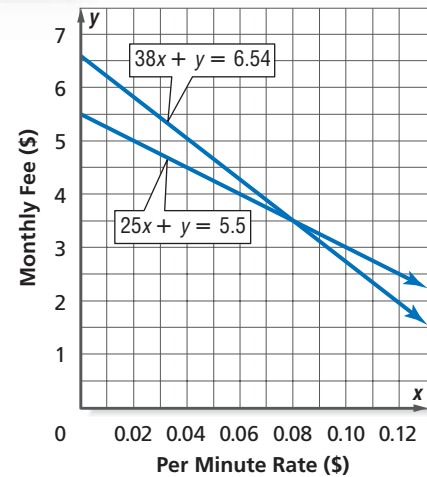
SUBSTITUTION   One algebraic method is the **substitution method**. Using this method, one equation is solved for one variable in terms of the other. Then, this expression is substituted for the variable in the other equation.

Example 1 Solve by Using Substitution

Use substitution to solve the system of equations.
$x + 2y = 8$
$\frac{1}{2}x - y = 18$

Solve the first equation for x in terms of y.

$x + 2y = 8$	First equation
$x = 8 - 2y$	Subtract 2y from each side.

Substitute $8 - 2y$ for x in the second equation and solve for y.

$\frac{1}{2}x - y = 18$	Second equation
$\frac{1}{2}(8 - 2y) - y = 18$	Substitute $8 - 2y$ for x.
$4 - y - y = 18$	Distributive Property
$-2y = 14$	Subtract 4 from each side.
$y = -7$	Divide each side by −2.

Now, substitute the value for y in either original equation and solve for x.

$x + 2y = 8$	First equation
$x + 2(-7) = 8$	Replace y with −7.
$x - 14 = 8$	Simplify.
$x = 22$	The solution of the system is (22, −7).

Study Tip

Coefficient of 1
It is easier to solve for the variable that has a coefficient of 1.

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Example 2 *Compare Values*

Quantitative Comparison Test Item

> Compare the quantity in Column A and the quantity in Column B. Then determine whether:
>
> Ⓐ the quantity in Column A is greater,
>
> Ⓑ the quantity in Column B is greater,
>
> Ⓒ the two quantities are equal, or
>
> Ⓓ the relationship cannot be determined from the information given.
>
> $$2x + y = 11$$
> $$x + 3y = 13$$
>
> | Column A | Column B |
> |:---:|:---:|
> | x | y |

Read the Test Item

You are asked to compare the values of x and y. Since this is a system of equations, you may be able to find the exact values for each variable.

Solve the Test Item

Step 1   Solve the first equation for y in terms of x since the coefficient of y is 1.

$2x + y = 11$ First equation

$\quad\quad y = 11 - 2x$ Subtract 2x from each side.

Step 2   Substitute $11 - 2x$ for y in the second equation.

$x + 3y = 13$ Second equation

$x - 3(11 - 2x) = 13$ Substitute 11 − 2x for y.

$x + 33 - 6x = 13$ Distributive Property

$-5x = -20$ Simplify.

$x = 4$ Divide each side by −5.

Step 3   Now replace x with 4 in either equation to find the value of y.

$2x + y = 11$ First equation

$2(4) + y = 11$ Substitute 4 for x.

$8 + y = 11$ Multiply.

$y = 3$ Subtract 8 from each side.

Step 4   Check the solution.

$2x + y = 11$ Original equation $x + 3y = 13$

$2(4) + 3 \overset{?}{=} 11$ Replace x with 4 and y with 3. $4 + 3(3) \overset{?}{=} 13$

$8 + 3 = 11 \checkmark$ Simplify. $4 + 9 = 13 \checkmark$

Step 5   Compare the values of x and y to answer the original problem.

$x = 4$ and $y = 3$

$4 > 3$

So, $x > y$.

The answer is A.

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ELIMINATION Another algebraic method is the **elimination method**. Using this method, you eliminate one of the variables by adding or subtracting the equations. When you add two true equations, the result is a new equation that is also true.

Example 3 Solve by Using Elimination

Use the elimination method to solve the system of equations.

$4a + 2b = 15$
$2a + 2b = 7$

In each equation, the coefficient of b is 2. If one equation is subtracted from the other, the variable b will be eliminated.

$$4a + 2b = 15$$
$$\underline{(-)\ 2a + 2b = 7}$$
$$2a \qquad = 8 \quad \text{Subtract the equations.}$$
$$a = 4 \quad \text{Divide each side by 2.}$$

Now find b by substituting 4 for a in either original equation.

$$2a + 2b = 7 \qquad \text{Second equation}$$
$$2(4) + 2b = 7 \qquad \text{Replace } a \text{ with 4.}$$
$$8 + 2b = 7 \qquad \text{Multiply.}$$
$$2b = -1 \qquad \text{Subtract 8 from each side.}$$
$$b = -\frac{1}{2} \qquad \text{Divide each side by 2.}$$

The solution is $\left(4, -\frac{1}{2}\right)$.

Sometimes, adding or subtracting the two equations will not eliminate either variable. You may use multiplication to write an equivalent equation so that one of the variables has the same or opposite coefficient in both equations. When you multiply an equation by a nonzero number, the new equation has the same set of solutions.

Example 4 Multiply, Then Use Elimination

Use the elimination method to solve the system of equations.

$3x - 7y = -14$
$5x + 2y = 45$

Multiply the first equation by 2 and the second equation by 7. Then add the equations to eliminate the y variable.

$3x - 7y = -14$ Multiply by 2. $6x - 14y = -28$

$5x + 2y = 45$ Multiply by 7. $\underline{(+)\ 35x + 14y = 315}$

$$41x \qquad = 287 \quad \text{Add the equations.}$$
$$x = 7 \qquad \text{Divide each side by 41.}$$

Replace x with 7 and solve for y.

$$3x - 7y = -14 \qquad \text{First equation}$$
$$3(7) - 7y = -14 \qquad \text{Replace } x \text{ with 7.}$$
$$21 - 7y = -14 \qquad \text{Multiply.}$$
$$-7y = -35 \qquad \text{Subtract 21 from each side.}$$
$$y = 5 \qquad \text{Divide each side by } -7.$$

The solution is $(7, 5)$.

◀ **CONTENTS** ▶

If you add or subtract two equations in a system and the result is an equation that is never true, then the system is inconsistent and it has no solution. If the result when you add or subtract two equations in a system is an equation that is always true, then the system is dependent and it has infinitely many solutions.

Example 5 Inconsistent System

Use the elimination method to solve the system of equations.
$$8x + 2y = 17$$
$$-4x - y = 9$$

Use multiplication to eliminate x.

$$8x + 2y = 17 \qquad\qquad\qquad 8x + 2y = 17$$
$$-4x - y = 9 \quad \boxed{\text{Multiply by 2.}} \quad \underline{-8x - 2y = 18}$$
$$0 = 35 \quad \text{Add the equations.}$$

Since there are no values of x and y that will make the equation $0 = 35$ true, there are no solutions for this system of equations.

Check for Understanding

Concept Check

1. **OPEN ENDED** Give an example of a system of equations that is more easily solved by substitution and a system that is more easily solved by elimination.

2. **Make a conjecture** about the solution of a system of equations if the result of subtracting one equation from the other is $0 = 0$.

3. **FIND THE ERROR** Juanita and Vincent are solving the system $2x - y = 6$ and $2x + y = 10$.

Juanita	Vincent
$2x - y = 6$	$2x - y = 6 \qquad 2x - y = 6$
$(-)2x + y = 10$	$(-)2x + y = 10 \qquad 2(4) - y = 6$
$0 = -4$	$4x \quad\;\; = 16 \qquad 8 - y = 6$
The statement $0 = -4$ is never true, so there is no solution.	$x = 4 \qquad\qquad y = 2$
	The solution is $(4,2)$.

Who is correct? Explain your reasoning.

Guided Practice

Solve each system of equations by using substitution.

4. $y = 3x - 4$
   $y = 4 + x$

5. $4c + 2d = 10$
   $c + 3d = 10$

Solve each system of equations by using elimination.

6. $2r - 3s = 11$
   $2r + 2s = 6$

7. $2p + 4q = 18$
   $3p - 6q = 3$

Solve each system of equations by using either substitution or elimination.

8. $a - b = 2$
   $-2a + 3b = 3$

9. $5m + n = 10$
   $4m + n = 4$

10. $3g - 2h = -1$
    $8h = 5 + 12g$

11. $\frac{1}{4}x + y = \frac{7}{2}$
    $x - \frac{1}{2}y = 2$

CONTENTS

12. QUANTITATIVE COMPARISON Compare the quantity in Column A and the quantity in Column B. Then determine whether:

Ⓐ the quantity in Column A is greater,

Ⓑ the quantity in Column B is greater,

Ⓒ the two quantities are equal, or

Ⓓ the relationship cannot be determined from the information given.

$$4x + 3y = 7$$
$$2x + y = 1$$

Column A	Column B
$2x + 2y$	6

Practice and Apply

Solve each system of equations by using substitution.

13. $2j - 3k = 3$
$j + k = 14$

14. $2r + s = 11$
$6r - 2s = -2$

15. $5a - b = 17$
$3a + 2b = 5$

16. $-w - z = -2$
$4w + 5z = 16$

17. $6c + 3d = 12$
$2c = 8 - d$

18. $2x + 4y = 6$
$7x = 4 + 3y$

Solve each system of equations by using elimination.

19. $u + v = 7$
$2u + v = 11$

20. $m - n = -9$
$7m + 2n = 9$

21. $3p - 5q = 6$
$2p - 4q = 4$

22. $4x - 5y = 17$
$3x + 4y = 5$

23. $2c + 6d = 14$
$\frac{1}{2}c - 3d = 8$

24. $3s + 2t = -3$
$s + \frac{1}{3}t = -4$

Solve each system of equations by using either substitution or elimination.

25. $r + 4s = -8$
$3r + 2s = 6$

26. $10m - 9n = 15$
$5m - 4n = 10$

27. $3c - 7d = -3$
$2c + 6d = -34$

28. $6g - 8h = 50$
$4g + 6h = 22$

29. $2p = 7 + q$
$6p - 3q = 24$

30. $3x = -31 + 2y$
$5x + 6y = 23$

31. $3u + 5v = 6$
$2u - 4v = -7$

32. $3a - 2b = -3$
$3a + b = 3$

33. $s + 3t = 27$
$\frac{1}{2}s + 2t = 19$

34. $f = 6 - 2g$
$\frac{1}{6}f + \frac{1}{3}g = 1$

35. $0.25x + 1.75y = 1.25$
$0.5x + 2.5y = 2$

36. $0.4m + 1.8n = 8$
$1.2m + 3.4n = 16$

37. Three times one number added to five times another number is 54. The second number is two less than the first. Find the numbers.

38. The average of two numbers is 7. Find the numbers if three times one of the numbers is one half the other number.

SKIING **For Exercises 39 and 40, use the following information.**
All 28 members in Crestview High School's Ski Club went on a one-day ski trip. Members can rent skis for $16.00 per day or snowboards for $19.00 per day. The club paid a total of $478 for rental equipment.

39. Write a system of equations that represents the number of members who rented the two types of equipment.

40. How many members rented skis and how many rented snowboards?

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41. HOUSING Campus Rentals rents 2- and 3-bedroom apartments for $700 and $900 per month, respectively. Last month they had six vacant apartments and reported $4600 in lost rent. How many of each type of apartment were vacant?

42. GEOMETRY Find the coordinates of the vertices of the parallelogram whose sides are contained in the lines whose equations are $2x + y = -12$, $2x - y = -8$, $2x - y - 4 = 0$, and $4x + 2y = 24$.

INVENTORY For Exercises 43 and 44, use the following information.
Heung-Soo is responsible for checking a shipment of technology equipment that contains laser printers that cost $700 each and color monitors that cost $200 each. He counts 30 boxes on the loading dock. The invoice states that the order totals $15,000.

43. Write a system of two equations that represents the number of each item.

44. How many laser printers and how many color monitors were delivered?

TEACHING For Exercises 45–47, use the following information.
Mr. Talbot is writing a test for his science classes. The test will have true/false questions worth 2 points each and multiple-choice questions worth 4 points each for a total of 100 points. He wants to have twice as many multiple-choice questions as true/false.

45. Write a system of equations that represents the number of each type of question.

46. How many true/false questions and multiple-choice questions will be on the test?

47. If most of his students can answer true/false questions within 1 minute and multiple-choice questions within $1\frac{1}{2}$ minutes, will they have enough time to finish the test in 45 minutes?

EXERCISE For Exercises 48 and 49, use the following information.
Megan exercises every morning for 40 minutes. She does a combination of step aerobics, which burns about 11 Calories per minute, and stretching, which burns about 4 Calories per minute. Her goal is to burn 335 Calories during her routine.

48. Write a system of equations that represents Megan's morning workout.

49. How long should she participate in each activity in order to burn 335 Calories?

50. CRITICAL THINKING Solve the system of equations.

$$\frac{1}{x} + \frac{3}{y} = \frac{3}{4}$$

$$\frac{3}{x} - \frac{2}{y} = \frac{5}{12}$$

$\left(\textit{Hint: Let } m = \frac{1}{x} \text{ and } n = \frac{1}{y}.\right)$

51. WRITING IN MATH   Answer the question that was posed at the beginning of the lesson.

How can a system of equations be used to make consumer decisions?

Include the following in your answer:

• a solution of the system of equations in the application at the beginning of the lesson, and

• an explanation of how Yolanda can use a graph to decide whether she should change to a long-distance plan that charges $0.10 per minute and a flat fee of $3.00 per month.
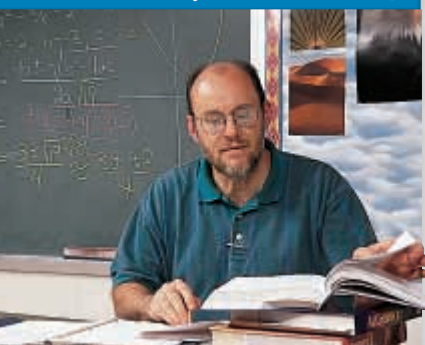
52. If $x = y + z$ and $x + y = 6$ and $x = 10$, then $z =$

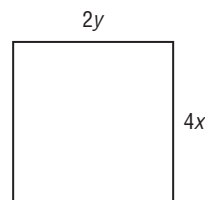   Ⓐ 4.   Ⓑ 8.   Ⓒ 14.   Ⓓ 16.

53. If the perimeter of the square shown at the right is 48 units, find the value of x.

   Ⓐ 3   Ⓑ 4

   Ⓒ 6   Ⓓ 8

2y (top of square), *4x* (right side of square)

Maintain Your Skills

Mixed Review   **Graph each system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.** *(Lesson 3-1)*

54. $y = x + 2$
$y = x - 1$

55. $4y - 2x = 4$
$y - \frac{1}{2}x = 1$

56. $3x + y = 1$
$y = 2x - 4$

Graph each inequality. *(Lesson 2-7)*

57. $x + y \le 3$

58. $5y - 4x < -20$

59. $3x + 9y \ge -15$

Write each equation in standard form. Identify A, B, and C. *(Lesson 2-2)*

60. $y = 7x + 4$

61. $x = y$

62. $3x = 2 - 5y$

63. $6x = 3y - 9$

64. $y = \frac{1}{2}x - 3$

65. $\frac{2}{3}y - 6 = 1 - x$

66. ELECTRICITY   Use the formula $I = \dfrac{E}{R + r}$ to find the amount of current I (in amperes) produced if the electromotive force E is 1.5 volts, the circuit resistance R is 2.35 ohms, and the resistance r within a battery is 0.15 ohms. *(Lesson 1-1)*

Getting Ready for the Next Lesson   **PREREQUISITE SKILL**   **Determine whether the given point satisfies each inequality.** *(To review **inequalities**, see Lesson 2-7.)*

67. $3x + 2y \le 10$; $(2, -1)$

68. $4x - 2y > 6$; $(3, 3)$

69. $7x + 4y \ge -15$; $(-4, 2)$

70. $7y + 6x < 50$; $(-5, 5)$

Practice Quiz 1
Lessons 3-1 and 3-2

Solve each system of equations by graphing. *(Lesson 3-1)*

1. $y = 3x + 10$
$y = -x + 6$

2. $2x + 3y = 12$
$2x - y = 4$

Solve each system of equations by using either substitution or elimination. *(Lesson 3-2)*

3. $y = x + 5$
$x + y = 9$

4. $2x + 6y = 2$
$3x + 2y = 10$

5. AIRPORTS   According to the Airports Council International, the busiest airport in the world is Atlanta's Hartsfield International Airport, and the second busiest is Chicago's O'Hare Airport. Together they handled 150.5 million passengers in the first six months of 1999. If Hartsfield handled 5.5 million more passengers than O'Hare, how many were handled by each airport? *(Lesson 3-2)*

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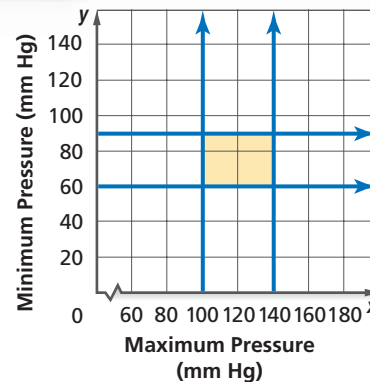# Solving Systems of Inequalities by Graphing

What You'll Learn

- Solve systems of inequalities by graphing.
- Determine the coordinates of the vertices of a region formed by the graph of a system of inequalities.

Vocabulary
- system of inequalities

How can you determine whether your blood pressure is in a normal range?

During one heartbeat, blood pressure reaches a maximum pressure (systolic) and a minimum pressure (diastolic), which are measured in millimeters of mercury (mm Hg). Blood pressure is expressed as the maximum pressure over the minimum pressure—for example, 120/80. Normal blood pressure for people under 40 ranges from 100 to 140 mm Hg for the maximum and from 60 to 90 mm Hg for the minimum. This information can be represented by a system of inequalities.

GRAPH SYSTEMS OF INEQUALITIES To solve a **system of inequalities**, we need to find the ordered pairs that satisfy all of the inequalities in the system. One way to solve a system of inequalities is to graph the inequalities on the same coordinate plane. The solution set is represented by the intersection of the graph.
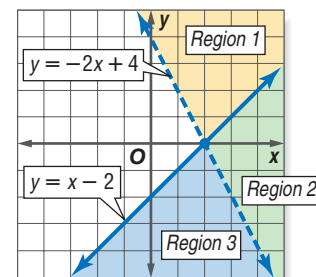
Example 1 Intersecting Regions

Solve each system of inequalities by graphing.

a. $y > -2x + 4$
$y \leq x - 2$

solution of $y > -2x + 4 \rightarrow$ Regions 1 and 2
solution of $y \leq x - 2 \quad \rightarrow$ Regions 2 and 3

The intersection of these regions is Region 2, which is the solution of the system of inequalities. Notice that the solution is a region containing an infinite number of ordered pairs.
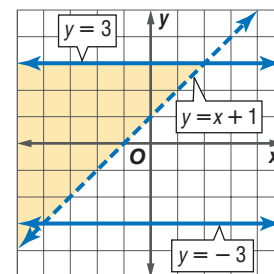
The green area represents where the yellow area of one graph overlaps the blue area of the other.

b. $y > x + 1$
$\left| y \right| \leq 3$

The inequality $\left| y \right| \leq 3$ can be written as $y \leq 3$ and $y \geq -3$.

Graph all of the inequalities on the same coordinate plane and shade the region or regions that are common to all.

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Study Tip

Reading Math
The empty set is also called the *null set*. It can be represented as $\varnothing$ or { }.

It is possible that two regions do *not* intersect. In such cases, we say the solution is the empty set $\varnothing$ and no solution exists.
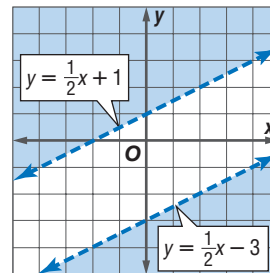
Example 2 Separate Regions

Solve the system of inequalities by graphing.

$$y > \frac{1}{2}x + 1$$

$$y < \frac{1}{2}x - 3$$

Graph both inequalities. The graphs do not overlap, so the solutions have no points in common. The solution set is $\varnothing$.

Example 3 Write and Use a System of Inequalities

SPACE EXPLORATION When NASA chose the first astronauts in 1959, size was important since the space available inside the Mercury capsule was very limited. NASA wanted men who were at least 5 feet 4 inches, but no more than 5 feet 11 inches tall, and who were between 21 and 40 years of age. Write and graph a system of inequalities that represents the range of heights and ages for qualifying astronauts.
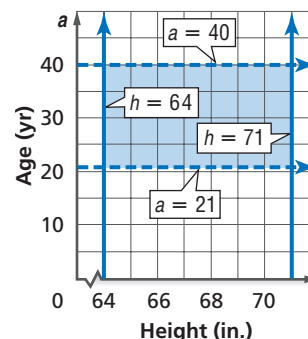
Let h represent the height of an astronaut in inches. The acceptable heights are at least 5 feet 4 inches (or 64 inches) and no more than 5 feet 11 inches (or 71 inches). We can write this information as two inequalities.

$$64 \leq h \text{ and } h \leq 71$$

Let a represent the age of an astronaut. The acceptable ages can also be written as two inequalities.

$$a > 21 \text{ and } a < 40$$

Graph all of the inequalities. Any ordered pair in the intersection of the graphs is a solution of the system.

More About. . .

Space Exploration

Today the basic physical qualifications for an astronaut are:

• blood pressure must be no greater than 140 over 90,
• distant visual acuity no greater than 20/100 uncorrected, correctable to 20/20, and
• height can range from 60 inches to 76 inches.

Source: NASA

FIND VERTICES OF A POLYGONAL REGION Sometimes, the graph of a system of inequalities forms a polygonal region. You can find the vertices of the region by determining the coordinates of the points at which the boundary lines intersect.

Example 4 Find Vertices

Find the coordinates of the vertices of the figure formed by $x + y \geq -1$, $x - y \leq 6$, and $12y + x \leq 32$.

Graph each inequality. The intersection of the graphs forms a triangle.

The coordinates $(-4, 3)$ and $(8, 2)$ can be determined from the graph. To find the coordinates of the third vertex, solve the system of equations $x + y = -1$ and $x - y = 6$.

CONTENTS
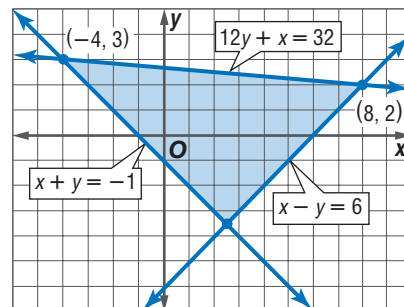
Add the equations to eliminate y.

$$x + y = -1$$
$$\underline{(+)\ x - y =\ \ \ 6}$$
$$2x\qquad =\ \ \ 5 \qquad \text{Add the equations.}$$
$$x = \frac{5}{2} \qquad \text{Divide each side by 2.}$$

Now find y by substituting $\frac{5}{2}$ for x in the first equation.

$$x + y = -1 \qquad \text{First equation}$$
$$\frac{5}{2} + y = -1 \qquad \text{Replace } x \text{ with } \frac{5}{2}.$$
$$y = -\frac{7}{2} \qquad \text{Subtract } \frac{5}{2} \text{ from each side.}$$

The vertices of the triangle are at $(-4, 3)$, $(8, 2)$, and $\left(\frac{5}{2}, -\frac{7}{2}\right)$.

# Check for Understanding

Concept Check

1. **OPEN ENDED** Write a system of inequalities that has no solution.

2. **Tell** whether the following statement is *true* or *false*. If false, give a counterexample. *A system of two linear inequalities has either no points or infinitely many points in its solution.*

3. **State** which region is the solution of the following systems of inequalities.

   a. $y \geq x$
       $y \leq -x$

   b. $y \leq x$
       $y \geq -x$

   c. $y \geq x$
       $y \geq -x$

   d. $y \leq x$
       $y \leq -x$
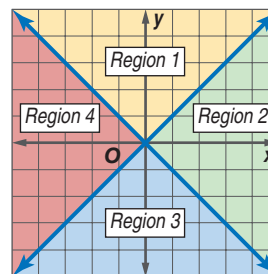
Guided Practice

Solve each system of inequalities by graphing.

4. $x \leq 4$
   $y > 2$

5. $y \geq x - 2$
   $y \leq -2x + 4$

6. $|x - 1| \leq 2$
   $x + y > 2$

7. $x \leq 1$
   $y < 2x + 1$
   $x + 2y \geq -3$

Find the coordinates of the vertices of the figure formed by each system of inequalities.

8. $y \leq x$
   $y \geq -3$
   $3y + 5x \leq 16$

9. $y \geq x - 3$
   $y \leq x + 7$
   $x + y \leq 11$
   $x + y \geq -1$

Application

SHOPPING For Exercises 10 and 11, use the following information.
Willis has been sent to the grocery store to purchase bagels and muffins for the members of the track team. He can spend at most $28. A package of bagels costs $2.50 and contains 6 bagels. A package of muffins costs $3.50 and contains 8 muffins. He needs to buy at least 12 bagels and 24 muffins.

10. Graph the region that shows how many packages of each item he can purchase.

11. Give an example of three different purchases he can make.

Lesson 3-3 Solving Systems of Inequalities by Graphing **125**

CONTENTS

Practice and Apply

Solve each system of inequalities by graphing.

12. $x \geq 2$
$y > 3$

13. $x \leq -1$
$y \geq -4$

14. $y < 2 - x$
$y > x + 4$

15. $y > x - 3$
$|y| \leq 2$

16. $3x + 2y \geq 6$
$4x - y \geq 2$

17. $4x - 3y < 7$
$2y - x < -6$

18. $y < 2x - 3$
$y \leq \frac{1}{2}x + 1$

19. $3y \leq 2x - 8$
$y \geq \frac{2}{3}x - 1$

20. $|x| \leq 3$
$|y| > 1$

21. $|x + 1| \leq 3$
$x + 3y \geq 6$

22. $y \geq 2x + 1$
$y \leq 2x - 2$
$3x + y \geq 9$

23. $x - 3y > 2$
$2x - y < 4$
$2x + 4y \geq -7$

Find the coordinates of the vertices of the figure formed by each system of inequalities.

24. $y \geq 0$
$x \geq 0$
$x + 2y \leq 8$

25. $y \geq -4$
$y \leq 2x + 2$
$2x + y \leq 6$

26. $x \leq 3$
$-x + 3y \leq 12$
$4x + 3y \geq 12$

27. $x + y \leq 9$
$x - 2y \leq 12$
$y \leq 2x + 3$

28. $y \geq -3$
$x \leq 6$
$y \geq x - 2$
$2y \leq x + 5$

29. $y \geq x - 5$
$y \leq 2x + 11$
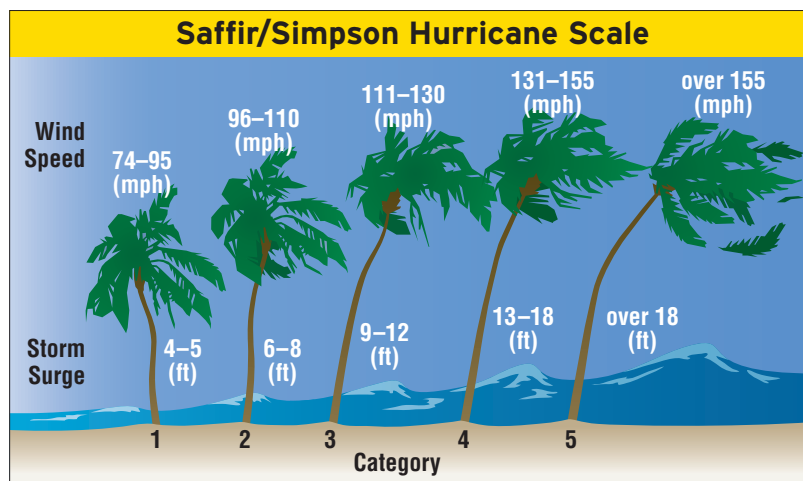$x + 2y \leq 12$
$x + 2y \geq 2$

30. Find the area of the region defined by the system of inequalities $y + x \leq 3$, $y - x \leq 3$, and $y \geq -1$.

31. Find the area of the region defined by the system of inequalities $x \geq -3$, $y + x \leq 8$, and $y - x \geq -2$.

32. **PART-TIME JOBS** Bryan Clark makes $10 an hour cutting grass and $12 an hour for raking leaves. He cannot work more than 15 hours per week. Graph two inequalities that Bryan can use to determine how many hours he needs to work at each job if he wants to earn at least $120 per week.

HURRICANES For Exercises 33 and 34, use the following information.
Hurricanes are divided into categories according to their wind speed and storm surge.

Saffir/Simpson Hurricane Scale

Category	1	2	3	4	5
Wind Speed	74–95 (mph)	96–110 (mph)	111–130 (mph)	131–155 (mph)	over 155 (mph)
Storm Surge	4–5 (ft)	6–8 (ft)	9–12 (ft)	13–18 (ft)	over 18 (ft)

33. Write and graph the system of inequalities that represents the range of wind speeds s and storm surges h for a category 3 hurricane.

34. On September 16, 1999, Hurricane Floyd hit the United States with winds of 140 mph. Classify Hurricane Floyd, and identify the heights of its storm surges.

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BAKING For Exercises 35–37, use the recipes at the right.

The Merry Bakers are baking pumpkin bread and Swedish soda bread for this week's specials. They have 24 cups of flour and at most 26 teaspoons of baking powder.

35. Graph the inequalities that represent how many loaves of each type of bread the bakers can make.

36. List three different combinations of breads they can make.

37. Which combination uses all of the available flour and baking soda?

38. **CRITICAL THINKING** Find the area of the region defined by $|x| + |y| \leq 5$ and $|x| + |y| \geq 2$.

39. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How can you determine whether your blood pressure is in a normal range?

Include the following in your answer:
- an explanation of how to use the graph, and
- a description of the regions that indicate high blood pressure, both systolic and diastolic.

Pumpkin Bread

2 c. of flour

1 tsp. baking powder

Swedish Soda Bread

$1\frac{1}{2}$ c. of flour

$2\frac{1}{2}$ tsp. baking powder

Standardized Test Practice
Ⓐ Ⓑ Ⓒ Ⓓ

40. Choose the system of inequalities whose solution is represented by the graph.

Ⓐ $y < -2$
     $x < -3$

Ⓑ $y \leq -2$
     $x > -3$

Ⓒ $x \leq -2$
     $y > -3$

Ⓓ $x < -3$
     $y < -3$
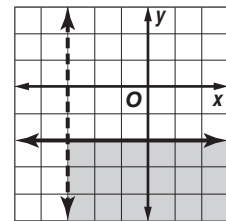
41. **OPEN ENDED** Create a system of inequalities for which the graph will be a square with its interior located in the first quadrant.

Maintain Your Skills

Mixed Review **Solve each system of equations by using either substitution or elimination.** *(Lesson 3-2)*

42. $4x - y = -20$
       $x + 2y = 13$

43. $3x - 4y = -2$
       $5x + 2y = 40$

44. $4x + 5y = 7$
       $3x - 2y = 34$

Solve each system of equations by graphing. *(Lesson 3-1)*

45. $y = 2x + 1$
       $y = -\frac{1}{2}x - 4$

46. $2x + y = -3$
       $6x + 3y = -9$

47. $2x - y = 6$
       $-x + 8y = 12$

48. Write an equation in slope-intercept form of the line that passes through $(-4, 4)$ and $(6, 9)$. *(Lesson 2-4)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find each value if $f(x) = 4x + 3$ and $g(x) = 5x - 7$.
*(To review **functions**, see Lesson 2-1.)*

49. $f(-2)$

50. $g(-1)$

51. $g(3)$

52. $f(6)$

53. $f(0.5)$

54. $g(-0.25)$

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Graphing Calculator Investigation

A Follow-Up of Lesson 3-3

Systems of Linear Inequalities

You can graph systems of linear inequalities with a TI-83 Plus calculator using the Y= menu. You can choose different graphing styles to shade above or below a line.

Example Graph the system of inequalities in the standard viewing window.

$y \geq -2x + 3$
$y \leq x + 5$

Step 1
- Enter $-2x + 3$ as Y1. Since y is greater than $-2x + 3$, shade above the line.

  KEYSTROKES: -2 [X,T,θ,n] [+] 3

- Use the left arrow key to move your cursor as far left as possible. Highlight the graph style icon. Press [ENTER] until the shade above icon, ▼, appears.

Step 2
- Enter $x + 5$ as Y2. Since y is less than $x + 5$, shade below the line.

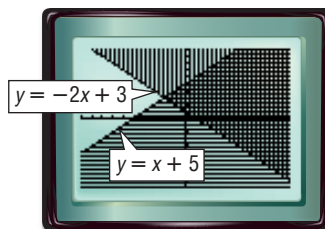 **KEYSTROKES:** [X,T,θ,n] [+] 5

- Use the arrow and [ENTER] keys to choose the shade below icon, ▶.

Step 3
- Display the graphs by pressing [GRAPH].

  Notice the shading pattern above the line $y = -2x + 3$ and the shading pattern below the line $y = x + 5$. The intersection of the graphs is the region where the patterns overlap. This region includes all the points that satisfy the system $y \geq -2x + 3$ and $y \leq x + 5$.

$y = -2x + 3$
$y = x + 5$

[−10, 10] scl: 1 by [−10, 10] scl: 1

Exercises

Solve each system of inequalities. Sketch each graph on a sheet of paper.

1. $y \geq 4$
$y \leq -x$

2. $y \geq -2x$
$y \leq -3$

3. $y \geq 1 - x$
$y \leq x + 5$

4. $y \geq x + 2$
$y \leq -2x - 1$

5. $3y \geq 6x - 15$
$2y \leq -x + 3$

6. $y + 3x \geq 6$
$y - 2x \leq 9$

7. $6y + 4x \geq 12$
$5y - 3x \leq -10$

8. $\frac{1}{4}y - x \geq -2$
$\frac{1}{3}y + 2x \leq 4$

www.algebra2.com/other_calculator_keystrokes

CONTENTS

3-4 Linear Programming

What You'll Learn

- Find the maximum and minimum values of a function over a region.
- Solve real-world problems using linear programming.

Vocabulary

- constraints
- feasible region
- bounded
- vertices
- unbounded
- linear programming

## How is linear programming used in scheduling work?

One of the primary tasks of the U.S. Coast Guard is to maintain the buoys that ships use to navigate. The ships that service buoys are called buoy tenders. They check the buoys in their area, make repairs, and replace any damaged buoys.

Suppose a certain buoy tender can carry up to 8 new buoys for making replacements. Their crew can check and repair a buoy in one hour. It takes the crew $2\frac{1}{2}$ hours to replace a buoy. The captain can use linear programming to find the maximum number of buoys this buoy tender can repair or replace in 24 hours at sea.

MAXIMUM AND MINIMUM VALUES The buoy tender captain can use a system of inequalities to represent the limitations of time and the number of replacement buoys on the ship. If these inequalities are graphed, all of the points in the intersection are the combinations of repairs and replacements that the buoy tender can schedule. The inequalities are called the **constraints**. The intersection of the graphs is called the **feasible region**. When the graph of a system of constraints is a polygonal region like the one graphed at the right, we say that the region is **bounded**.

Sometimes it is necessary to find the maximum or minimum values that a linear function has for the points in a feasible region. For example, the buoy tender captain wishes to maximize the total number of buoys serviced. The maximum or minimum value of a related function *always* occurs at one of the **vertices** of the feasible region.
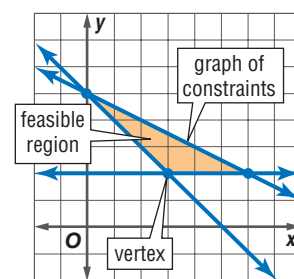
Example 1 Bounded Region

Graph the following system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function $f(x, y) = 3x + y$ **for this region.**

$$x \geq 1$$
$$y \geq 0$$
$$2x + y \leq 6$$

Step 1 Find the vertices of the region. Graph the inequalities.

The polygon formed is a triangle with vertices at $(1, 4)$, $(3, 0)$, and $(1, 0)$.
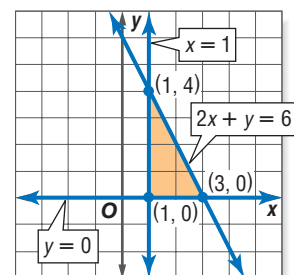
Study Tip

Reading Math
The notation $f(x, y)$ is used to represent a function with two variables x and y. It is read *f of x and y*.

CONTENTS

Step 2 Use a table to find the maximum and minimum values of $f(x, y)$.
Substitute the coordinates of the vertices into the function.

(x, y)	3x + y	f(x, y)	
(1, 4)	3(1) + 4	7	
(3, 0)	3(3) + 0	9	← maximum
(1, 0)	3(1) + 0	3	← minimum

The maximum value is 9 at (3, 0). The minimum value is 3 at (1, 0).

Sometimes a system of inequalities forms a region that is open. In this case, the region is said to be **unbounded**.

Example 2 Unbounded Region

Graph the following system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function $f(x, y) = 5x + 4y$ for this region.

$2x + y \geq 3$
$3y - x \leq 9$
$2x + y \leq 10$

Graph the system of inequalities. There are only two points of intersection, (0, 3) and (3, 4).

(x, y)	5x + 4y	f(x, y)
(0, 3)	5(0) + 4(3)	12
(3, 4)	5(3) + 4(4)	31

The maximum is 31 at (3, 4).

Although $f(0, 3)$ is 12, it is not the minimum value since there are other points in the solution that produce lesser values. For example, $f(3, 2) = 7$ and $f(20, -35) = -40$. It appears that because the region is unbounded, $f(x, y)$ has no minimum value.
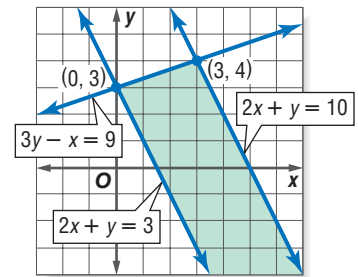
Study Tip

Common Misconception
Always test a point contained in the feasible region when the graph is unbounded. Do not assume that there is no minimum value if the feasible region is unbounded below the line, or that there is no maximum value if the feasible region is unbounded above the line.

REAL-WORLD PROBLEMS The process of finding maximum or minimum values of a function for a region defined by inequalities is called **linear programming**. The steps used to solve a problem using linear programming are listed below.

Key Concept   *Linear Programming Procedure*

Step 1 Define the variables.

Step 2 Write a system of inequalities.

Step 3 Graph the system of inequalities.

Step 4 Find the coordinates of the vertices of the feasible region.

Step 5 Write a function to be maximized or minimized.

Step 6 Substitute the coordinates of the vertices into the function.

Step 7 Select the greatest or least result. Answer the problem.

Linear programming can be used to solve many types of real-world problems. These problems have certain restrictions placed on the variables, and some function of the variable must be maximized or minimized.

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Example 3 Linear Programming

VETERINARY MEDICINE As a receptionist for a veterinarian, one of Dolores Alvarez's tasks is to schedule appointments. She allots 20 minutes for a routine office visit and 40 minutes for a surgery. The veterinarian cannot do more than 6 surgeries per day. The office has 7 hours available for appointments. If an office visit costs $55 and most surgeries cost $125, find a combination of office visits and surgeries that will maximize the income the veterinarian practice receives per day.

More About...

Veterinary Medicine

Surgeries are usually performed in the morning so that the animal can recover throughout the day while there is plenty of staff to monitor its progress.

Source: www.vetmedicine. miningco.com

Step 1 Define the variables.

v = the number of office visits
s = the number of surgeries

Step 2 Write a system of inequalities.

Since the number of appointments cannot be negative, v and s must be nonnegative numbers.

$v \geq 0$ and $s \geq 0$

An office visit is 20 minutes, and a surgery is 40 minutes. There are 7 hours available for appointments.

$20v + 40s \leq 420$ 7 hours = 420 minutes

The veterinarian cannot do more than 6 surgeries per day.

$s \leq 6$

Step 3 Graph the system of inequalities.

Step 4 Find the coordinates of the vertices of the feasible region.

From the graph, the vertices of the feasible region are at $(0, 0)$, $(6, 0)$, $(6, 9)$, and $(0, 21)$. If the vertices could not be read from the graph easily, we could also solve a system of equations using the boundaries of the inequalities.
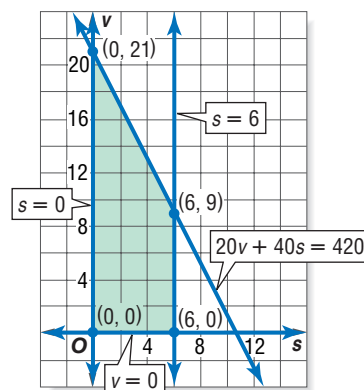
Step 5 Write a function to be maximized or minimized.

The function that describes the income is $f(s, v) = 125s + 55v$. We wish to find the maximum value for this function.

Step 6 Substitute the coordinates of the vertices into the function.

(s, v)	$125s + 55v$	$f(s, v)$
$(0, 0)$	$125(0) + 55(0)$	0
$(6, 0)$	$125(6) + 55(0)$	750
$(6, 9)$	$125(6) + 55(9)$	1245
$(0, 21)$	$125(0) + 55(21)$	1155

Step 7 Select the greatest or least result. Answer the problem.

The maximum value of the function is 1245 at $(6, 9)$. This means that the maximum income is $1245 when Dolores schedules 6 surgeries and 9 office visits.

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Concept Check

1. **Determine** whether the following statement is *always*, *sometimes*, or *never* true.
   A feasible region has a minimum and a maximum value.

2. **OPEN ENDED** Give an example of a system of inequalities that forms a bounded region.

Guided Practice

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

3. $y \geq 2$
   $x \geq 1$
   $x + 2y \leq 9$
   $f(x, y) = 2x - 3y$

4. $x \geq -3$
   $y \leq 1$
   $3x + y \leq 6$
   $f(x, y) = 5x - 2y$

5. $y \leq 2x + 1$
   $1 \leq y \leq 3$
   $x + 2y \leq 12$
   $f(x, y) = 3x + y$

6. $y \geq -x + 2$
   $2 \leq x \leq 7$
   $y \leq \frac{1}{2}x + 5$
   $f(x, y) = 8x + 3y$

7. $x + 2y \leq 6$
   $2x - y \leq 7$
   $x \geq -2, y \geq -3$
   $f(x, y) = x - y$

8. $x - 3y \geq -7$
   $5x + y \leq 13$
   $x + 6y \geq -9$
   $3x - 2y \geq -7$
   $f(x, y) = x - y$

Application

MANUFACTURING For Exercises 9–14, use the following information.
The students in the Future Homemakers Club are making canvas tote bags and leather tote bags for a money making project. They will line both types of tote bags with canvas and use leather for the handles of both bags. For the canvas tote bags, they need 4 yards of canvas and 1 yard of leather. For the leather tote bags, they need 3 yards of leather and 2 yards of canvas. Their faculty advisor has purchased 56 yards of leather and 104 yards of canvas.

9. Let c represent the number of canvas tote bags and let ℓ represent the number of leather tote bags. Write a system of inequalities to represent the number of tote bags that can be produced.

10. Draw the graph showing the feasible region.

11. List the coordinates of the vertices of the feasible region.

12. If the club plans to sell the canvas bags at a profit of $20 each and the leather bags at a profit of $35 each, write a function for the total profit on the bags.

13. Determine the number of canvas and leather bags that they need to make for a maximum profit.

14. What is the maximum profit?

Homework Help

For Exercises	See Examples
15–29	1, 2
31–36, 38–42	3

Extra Practice

See page 833.

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

15. $y \geq 1$
    $x \leq 6$
    $y \leq 2x + 1$
    $f(x, y) = x + y$

16. $y \geq -4$
    $x \leq 3$
    $y \leq 3x - 4$
    $f(x, y) = x - y$

17. $y \geq 2$
    $1 \leq x \leq 5$
    $y \leq x + 3$
    $f(x, y) = 3x - 2y$

18. $y \geq 1$
    $2 \leq x \leq 4$
    $x - 2y \geq -4$
    $f(x, y) = 3y + x$

19. $y \leq x + 2$
    $y \leq 11 - 2x$
    $2x + y \geq -7$
    $f(x, y) = 4x - 3y$

20. $y \leq x + 6$
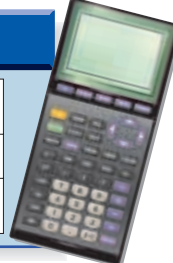 $y + 2x \geq 6$
    $2 \leq x \leq 6$
    $f(x, y) = -x + 3y$

CONTENTS

21. $x + y \leq 3$
$x + 2y \leq 4$
$x \geq 0, y \geq 0$
$f(x, y) = 3y - 4x$

22. $y \leq 7 - x$
$3x - 2y \leq 6$
$x \geq 0, y \geq 0$
$f(x, y) = 5x - 2y$

23. $y \geq x - 3$
$y \leq 6 - 2x$
$2x + y \geq -3$
$f(x, y) = 3x + 4y$

24. $x + y \geq 4$
$3x - 2y \leq 12$
$x - 4y \geq -16$
$f(x, y) = x - 2y$

25. $x + y \geq 2$
$4y \leq x + 8$
$y \geq 2x - 5$
$f(x, y) = 4x + 3y$

26. $2x + 2y \geq 4$
$2y \geq 3x - 6$
$4y \leq x + 8$
$f(x, y) = 3y + x$

27. $2x + 3y \geq 6$
$3x - 2y \geq -4$
$5x + y \geq 15$
$f(x, y) = x + 3y$

28. $x \geq 0$
$y \geq 0$
$x + 2y \leq 6$
$2y - x \leq 2$
$x + y \leq 5$
$f(x, y) = 3x - 5y$

29. $x \geq 2$
$y \geq 1$
$x - 2y \geq -4$
$x + y \leq 8$
$2x - y \leq 7$
$f(x, y) = x - 4y$

30. CRITICAL THINKING   The vertices of a feasible region are $A(1, 2)$, $B(5, 2)$, and $C(1, 4)$. Write a function that satisfies each condition.

a. A is the maximum and B is the minimum.
b. C is the maximum and B is the minimum.
c. B is the maximum and A is the minimum.
d. A is the maximum and C is the minimum.
e. B and C are both maxima and A is the minimum.

PRODUCTION   **For Exercises 31–36, use the following information.**
There are a total of 85 workers' hours available per day for production at a calculator manufacturer. There are 40 workers' hours available for encasement and quality control each day. The table below shows the number of hours needed in each department for two different types of calculators.

Calculator Production Time		
Calculator Type	**Production Time**	**Encasement and Quality Control**
graphing calculator	$1\frac{1}{2}$ hours	2 hours
computer-algebra systems (CAS)	1 hour	$\frac{1}{2}$ hour

31. Let g represent the number of graphing calculators and let c represent the number of CAS calculators. Write a system of inequalities to represent the number of calculators that can be produced.

32. Draw the graph showing the feasible region.

33. List the coordinates of the vertices of the feasible region.

34. If the profit on a graphing calculator is \$50 and the profit on a CAS calculator is \$65, write a function for the total profit on the calculators.

35. Determine the number of each type of calculator that is needed to make a maximum profit.

36. What is the maximum profit?

37. RESEARCH   Use the Internet or other reference to find an industry that uses linear programming. Describe the restrictions or constraints of the problem and explain how linear programming is used to help solve the problem.

CONTENTS

FARMING For Exercises 38–41, use the following information.

Dean Stadler has 20 days in which to plant corn and soybeans. The corn can be planted at a rate of 250 acres per day and the soybeans at a rate of 200 acres per day. He has 4500 acres available for planting these two crops.

38. Let c represent the number of acres of corn and let s represent the number of acres of soybeans. Write a system of inequalities to represent the possible ways Mr. Stadler can plant the available acres.

39. Draw the graph showing the feasible region and list the coordinates of the vertices of the feasible region.

40. If the profit on corn is $26 per acre and the profit on soybeans is $30 per acre, how much of each should Mr. Stadler plant? What is the maximum profit?

41. How much of each should Mr. Stadler plant if the profit on corn is $29 per acre and the profit on soybeans is $24 per acre? What is the maximum profit?

42. PACKAGING The Cookie Factory's best selling items are chocolate chip cookies and peanut butter cookies. They want to sell both types of cookies together in combination packages. The different-sized packages will contain between 6 and 12 cookies, inclusively. At least three of each type of cookie should be in each package. The cost of making a chocolate chip cookie is 19¢, and the selling price is 44¢ each. The cost of making a peanut butter cookie is 13¢, and the selling price is 39¢. How many of each type of cookie should be in each package to maximize the profit?

43. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How is linear programming used in scheduling work?

Include the following in your answer:
- a system of inequalities that represents the constraints that are used to schedule buoy repair and replacement,
- an explanation of the linear function that the buoy tender captain would wish to maximize, and
- a demonstration of how to solve the linear programming problem to find the maximum number of buoys the buoy tender could service in 24 hours at sea.

Standardized Test Practice
Ⓐ Ⓑ Ⓒ Ⓓ

44. A feasible region has vertices at $(0, 0)$, $(4, 0)$, $(5, 5)$, and $(0, 8)$. Find the maximum and minimum of the function $f(x, y) = x + 3y$ over this region.

Ⓐ maximum: $f(0, 8) = 24$
minimum: $f(0, 0) = 0$

Ⓑ minimum: $f(0, 0) = 0$
maximum: $f(5, 5) = 20$

Ⓒ maximum: $f(5, 5) = 20$
minimum: $f(0, 8) = 8$

Ⓓ minimum: $f(4, 0) = 4$
maximum: $f(0, 0) = 0$

45. What is the area of square $ABCD$?
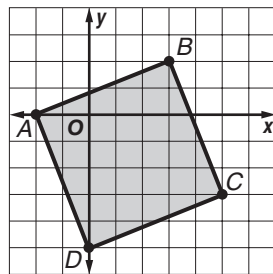
Ⓐ 25 units^2
Ⓑ $4\sqrt{29}$ units^2
Ⓒ 29 units^2
Ⓓ $25 + \sqrt{2}$ units^2

CONTENTS

Mixed Review

Solve each system of inequalities by graphing. *(Lesson 3-3)*

46. $2y + x \geq 4$
$y \geq x - 4$

47. $3x - 2y \leq -6$
$y \leq \frac{3}{2}x - 1$

Solve each system of equations by using either substitution or elimination. *(Lesson 3-2)*

48. $4x + 5y = 20$
$5x + 4y = 7$

49. $6x + y = 15$
$x - 4y = -10$

50. $3x + 8y = 23$
$5x - y = 24$

SCHOOLS For Exercises 51 and 52, use the graph at the right. *(Lesson 1-3)*

51. Define a variable and write an equation that can be used to determine on average how much the annual per-pupil spending has increased from 1986 to 2001.
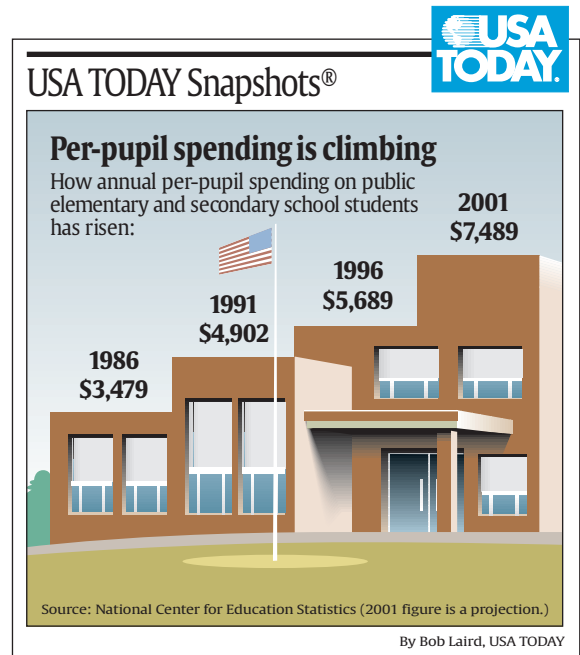
52. Solve the problem.

Name the property illustrated by each equation. *(Lesson 1-2)*

53. $4n + (-4n) = 0$

54. $(2 \cdot 5) \cdot 6 = 2 \cdot (5 \cdot 6)$

55. $\left(-\frac{3}{2}\right)\left(-\frac{2}{3}\right) = 1$

56. $6(x + 9) = 6x + 6(9)$

USA TODAY Snapshots®

Per-pupil spending is climbing
How annual per-pupil spending on public elementary and secondary school students has risen:

2001
$7,489

1996
$5,689

1991
$4,902

1986
$3,479

Source: National Center for Education Statistics (2001 figure is a projection.)

By Bob Laird, USA TODAY

Getting Ready for the Next Lesson

PREREQUISITE SKILL Evaluate each expression if $x = -2$, $y = 6$, and $z = 5$.
(To review evaluating expressions, see Lesson 1-1.)

57. $x + y + z$

58. $2x - y + 3z$

59. $-x + 4y - 2z$

60. $5x + 2y - z$

61. $3x - y + 4z$

62. $-2x - 3y + 2z$

Practice Quiz 2
Lessons 3-3 and 3-4

Solve each system of inequalities by graphing. *(Lesson 3-3)*

1. $y - x > 0$
$y + x < 4$

2. $y \geq 3x - 4$
$y \leq x + 3$

3. $x + 3y \geq 15$
$4x + y \leq 16$

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region. *(Lesson 3-4)*

4. $x \geq 0$
$y \geq 0$
$y \leq 2x + 4$
$3x + y \leq 9$
$f(x, y) = 2x + y$

5. $x \leq 5$
$y \geq -3x$
$2y \leq x + 7$
$y \geq x - 4$
$f(x, y) = 4x - 3y$

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Graphing Equations in Three Variables

To graph an equation in three variables, it is necessary to add a third dimension to our coordinate system. The graph of an equation of the form $Ax + By + Cz = D$, where A, B, C, and D can not all be equal to zero is a plane.

When graphing in three-dimensional space, begin with the xy-coordinate plane in a horizontal position. Then draw the z-axis as a vertical line passing through the origin. There are now three coordinate planes: the xy-plane, the xz-plane, and the yz-plane. These planes intersect at right angles and divide space into eight regions, called **octants**.

A point in space (three dimensions) has three coordinates and is represented by an **ordered triple** (x, y, z).
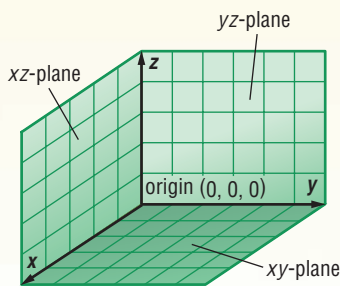
The first octant contains the points in space for which all three coordinates are positive.
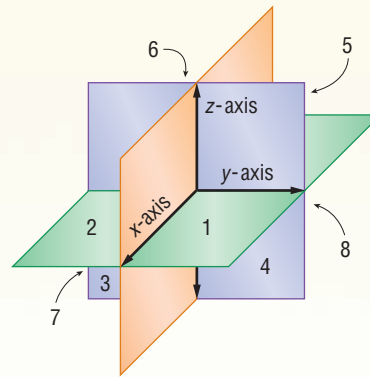
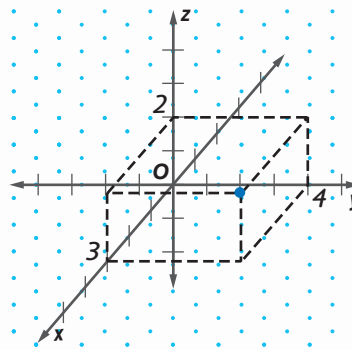The octants are numbered as shown.

Activity 1

Use isometric dot paper to graph (3, 4, 2) on a three-dimensional coordinate system. Name the octant in which it lies.

Draw the x-, y-, and z-axes as shown.

Begin by finding the point $(3, 4, 0)$ in the xy-plane.

The z-coordinate is 2, so move the point up two units parallel to the z-axis.

The point lies in octant 1.

To graph a linear equation in three variables, first find the intercepts of the graph. Connect the intercepts on each axis. This forms a portion of a plane that lies in a single octant.

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Activity 2

Graph $2x + 3y + 4z = 12$.

Begin by finding the x-, y-, and z-intercepts.

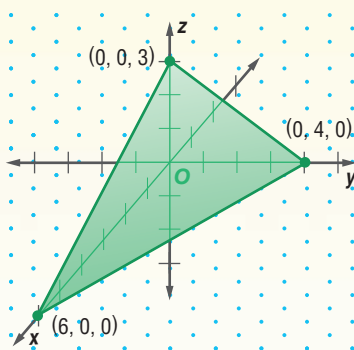| x-intercept	y-intercept	z-intercept
Let $y = 0$ and $z = 0$.	Let $x = 0$ and $z = 0$.	Let $x = 0$ and $y = 0$.
$2x = 12$	$3y = 12$	$4z = 12$
$x = 6$	$y = 4$	$z = 3$

To sketch the plane, graph the intercepts, which have coordinates $(6, 0, 0)$, $(0, 4, 0)$, and $(0, 0, 3)$. Then connect the points. Remember this is only a portion of the plane that extends indefinitely.

Model and Analyze

Graph each ordered triple on a three-dimensional coordinate system. Name the octant in which each point lies.

  1. $(5, 3, 6)$ **2.** $(-2, 4, 3)$ **3.** $(1, -5, 7)$

Graph each equation. Name the coordinates for the x-, y-, and z-intercepts.

  4. $3x + 6y + z = 6$ **5.** $2x - 5y + 4z = 20$ **6.** $x + 3y - 6z = 3$

  7. $-3x + 5y + 10z = 15$ **8.** $6x + 9z = 18$ **9.** $4x - 6y = 24$

Write an equation of the plane given its x-, y-, and z-intercepts, respectively.

10. $8, -3, 6$ **11.** $10, 4, -5$ **12.** $\frac{1}{2}, 4, -12$

13. Describe the values of x, y, and z as either positive or negative for each octant.

14. Consider the graph $x = -3$ in one, two, and three dimensions.
    a. Graph the equation on a number line.
    b. Graph the equation on a coordinate plane.
    c. Graph the equation in a three-dimensional coordinate axis.
    d. Describe and compare the graphs in parts **a**, **b**, and **c**.
    e. **Make a conjecture** about the graph of $x > -3$ in one, two, and three dimensions.

3-5 Solving Systems of Equations in Three Variables

What You'll Learn

- Solve systems of linear equations in three variables.
- Solve real-world problems using systems of linear equations in three variables.

How can you determine the number and type of medals U.S. Olympians won?

At the 2000 Summer Olympics in Sydney, Australia, the United States won 97 medals. They won 6 more gold medals than bronze and 8 fewer silver medals than bronze.

You can write and solve a system of three linear equations to determine how many of each type of medal the U.S. Olympians won. Let g represent the number of gold medals, let s represent the number of silver medals, and let b represent the number of bronze medals.

$g + s + b = 97$ The U.S. won a total of 97 medals.

$g = b + 6$ They won 6 more gold medals than bronze.

$s = b - 8$ They won 8 fewer silver medals than bronze.

SYSTEMS IN THREE VARIABLES   The system of equations above has three variables. The graph of an equation in three variables, all to the first power, is a plane. The solution of a system of three equations in three variables can have one solution, infinitely many solutions, or no solution.
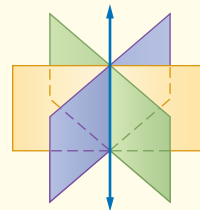
Key Concept — System of Equations in Three Variables
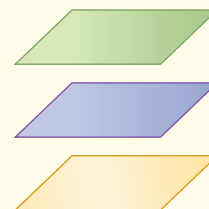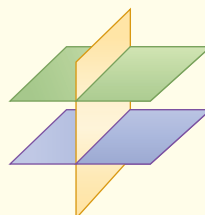
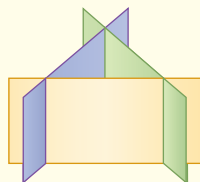**One Solution**
- planes intersect in one point

(x, y, z)
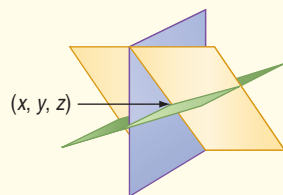
Infinite Solutions
- planes intersect in a line
- planes intersect in the same plane

No Solution
- planes have no point in common

CONTENTS

Solving systems of equations in three variables is similar to solving systems of equations in two variables. Use the strategies of substitution and elimination. The solution of a system of equations in three variables x, y, and z is called an **ordered triple** and is written as (x, y, z).

Example 1 One Solution

Solve the system of equations.

$x + 2y + z = 10$
$2x - y + 3z = -5$
$2x - 3y - 5z = 27$

Step 1 Use elimination to make a system of two equations in two variables.

$x + 2y + z = 10$ Multiply by 2.

$2x - y + 3z = -5$

$$2x + 4y + 2z = 20$$
$$(-)\ 2x - y + 3z = -5$$
$$\overline{5y - z = 25}$$ Subtract to eliminate x.

$$2x - y + 3z = -5 \quad \text{Second equation}$$
$$(-)\ 2x - 3y - 5z = 27 \quad \text{Third equation}$$
$$\overline{2y + 8z = -32} \quad \text{Subtract to eliminate } x.$$

Notice that the x terms in each equation have been eliminated. The result is two equations with the same two variables y and z.

Step 2 Solve the system of two equations.

$5y - z = 25$ Multiply by 8.

$2y + 8z = -32$

$$40y - 8z = 200$$
$$(+)\ 2y + 8z = -32$$
$$\overline{42y = 168} \quad \text{Add to eliminate } z.$$
$$y = 4 \quad \text{Divide by 42.}$$

Substitute 4 for y in one of the equations with two variables and solve for z.

$$5y - z = 25 \quad \text{Equation with two variables}$$
$$5(4) - z = 25 \quad \text{Replace } y \text{ with 4.}$$
$$20 - z = 25 \quad \text{Multiply.}$$
$$z = -5 \quad \text{Simplify.}$$

The result is $y = 4$ and $z = -5$.

Step 3 Substitute 4 for y and -5 for z in one of the original equations with three variables.

$$x + 2y + z = 10 \quad \text{Original equation with three variables}$$
$$x + 2(4) + (-5) = 10 \quad \text{Replace } y \text{ with 4 and } z \text{ with } -5.$$
$$x + 8 - 5 = 10 \quad \text{Multiply.}$$
$$x = 7 \quad \text{Simplify.}$$

The solution is $(7, 4, -5)$. You can check this solution in the other two original equations.

Example 2 Infinite Solutions

Solve the system of equations.

$4x - 6y + 4z = 12$
$6x - 9y + 6z = 18$
$5x - 8y + 10z = 20$

Eliminate x in the first two equations.

$4x - 6y + 4z = 12$	Multiply by 3.	$12x - 18y + 12z = 36$	
$6x - 9y + 6z = 18$	Multiply by -2.	$\underline{(+)\ -12x + 18y - 12z = -36}$	Add the equations.
		$0 = 0$	

The equation $0 = 0$ is always true. This indicates that the first two equations represent the same plane. Check to see if this plane intersects the third plane.

$4x - 6y + 4z = 12$	Multiply by 5.	$20x - 30y + 20z = 60$	
$5x - 8y + 10z = 20$	Multiply by -2.	$\underline{(+)\ -10x + 16y - 20z = -40}$	Add the equations.
		$10x - 14y \ = 20$	
		$5x - 7y \ = 10$	Divide by the GCF, 2.

The planes intersect in the line. So, there are an infinite number of solutions.

Example 3 No Solution

Solve the system of equations.

$6a + 12b - 8c = 24$
$9a + 18b - 12c = 30$
$4a + 8b - 7c = 26$

Eliminate a in the first two equations.

$6a + 12b - 8c = 24$	Multiply by 3.	$18a + 36b - 24c = 72$	
$9a + 18b - 12c = 30$	Multiply by 2.	$\underline{(-)\ 18a + 36b - 24c = 60}$	Subtract the equations.
		$0 = 12$	

The equation $0 = 12$ is never true. So, there is no solution of this system.

REAL-WORLD PROBLEMS   When solving problems involving three variables, use the four-step plan to help organize the information.

Example 4 Write and Solve a System of Equations

• **INVESTMENTS**   Andrew Chang has $15,000 that he wants to invest in certificates of deposit (CDs). For tax purposes, he wants his total interest per year to be $800. He wants to put $1000 more in a 2-year CD than in a 1-year CD and invest the rest in a 3-year CD. How much should Mr. Chang invest in each type of CD?

Number of Years	1	2	3
Rate	3.4%	5.0%	6.0%

Explore   Read the problem and define the variables.

a = the amount of money invested in a 1-year certificate

b = the amount of money in a 2-year certificate

c = the amount of money in a 3-year certificate

CONTENTS

Plan   Mr. Chang has $15,000 to invest.

$$a + b + c = 15,000$$

The interest he earns should be $800. The interest equals the rate times the amount invested.

$$0.034a + 0.05b + 0.06c = 800$$

There is $1000 more in the 2-year certificate than in the 1-year certificate.

$$b = a + 1000$$

Solve   Substitute $b = a + 1000$ in each of the first two equations.

$a + (a + 1000) + c = 15,000$	Replace b with ($a + 1000$).
$2a + 1000 + c = 15,000$	Simplify.
$2a + c = 14,000$	Subtract 1000 from each side.

$0.034a + 0.05(a + 1000) + 0.06c = 800$	Replace b with ($a + 1000$).
$0.034a + 0.05a + 50 + 0.06c = 800$	Distributive Property
$0.084a + 0.06c = 750$	Simplify.

Now solve the system of two equations in two variables.

$2a + c = 14,000$ Multiply by 0.06. $0.12a + 0.06c = 840$

$0.084a + 0.06c = 750$ $(-)\ 0.084a + 0.06c = 750$

$$0.036a = 90$$
$$a = 2500$$

Substitute 2500 for a in one of the original equations.

$b = a + 1000$	Third equation
$= 2500 + 1000$	$a = 2500$
$= 3500$	Add.

Substitute 2500 for a and 3500 for b in one of the original equations.

$a + b + c = 15,000$	First equation
$2500 + 3500 + c = 15,000$	$a = 2500, b = 3500$
$6000 + c = 15,000$	Add.
$c = 9000$	Subtract 6000 from each side.

So, Mr. Chang should invest $2500 in a 1-year certificate, $3500 in a 2-year certificate, and $9000 in a 3-year certificate.

Examine   Check to see if all the criteria are met.

The total investment is $15,000.

$2500 + 3500 + 9000 = 15,000$ ✓

The interest earned will be $800.

$0.034(2500) + 0.05(3500) + 0.06(9000) = 800$

$\quad\quad 85 \quad + \quad 175 \quad + \quad 540 \quad = 800$ ✓

There is $1000 more in the 2-year certificate than the 1-year certificate.

$3500 = 2500 + 1000$ ✓

CONTENTS

Check for Understanding

1. Explain how you can use the methods of solving a system of two equations in two variables to solve a system of three equations in three variables.

2. FIND THE ERROR Melissa is solving the system of equations $r + 2s + t = 3$, $2r + 4s + 2t = 6$, and $3r + 6s + 3t = 12$.

> $r + 2s + t = 3 \rightarrow \quad 2r + 4s + 2t = 6$
>
> $2r + 4s + 2t = 6 \rightarrow \underline{(-)2r + 4s + 2t = 6}$
>
> $\qquad\qquad\qquad\qquad\qquad 0 = 0$
>
> The second equation is a multiple of the first, so they are the same plane. There are infinitely many solutions.

Is she correct? Explain your reasoning.

3. OPEN ENDED Give an example of a system of three equations in three variables that has $(-3, 5, 2)$ as a solution. Show that the ordered triple satisfies all three equations.

Solve each system of equations.

4. $x + 2y = 12$
$3y - 4z = 25$
$x + 6y + z = 20$

5. $9a + 7b = -30$
$8b + 5c = 11$
$-3a + 10c = 73$

6. $r - 3s + t = 4$
$3r - 6s + 9t = 5$
$4r - 9s + 10t = 9$

7. $2r + 3s - 4t = 20$
$4r - s + 5t = 13$
$3r + 2s + 4t = 15$

8. $2x - y + z = 1$
$x + 2y - 4z = 3$
$4x + 3y - 7z = -8$

9. $x + y + z = 12$
$6x - 2y - z = 16$
$3x + 4y + 2z = 28$

COOKING **For Exercises 10 and 11, use the following information.**
Jambalaya is a Cajun dish made from chicken, sausage, and rice. Simone is making a large pot of jambalaya for a party. Chicken costs $6 per pound, sausage costs $3 per pound, and rice costs $1 per pound. She spends $42 on $13\frac{1}{2}$ pounds of food. She buys twice as much rice as sausage.

10. Write a system of three equations that represents how much food Simone purchased.

11. How much chicken, sausage, and rice will she use in her dish?

Practice and Apply

For Exercises	See Examples
12–23	1–3
24–30	4

See page 833.

Solve each system of equations.

12. $2x - y = 2$
$3z = 21$
$4x + z = 19$

13. $-4a = 8$
$5a + 2c = 0$
$7b + 3c = 22$

14. $5x + 2y = 4$
$3x + 4y + 2z = 6$
$7x + 3y + 4z = 29$

15. $8x - 6z = 38$
$2x - 5y + 3z = 5$
$x + 10y - 4z = 8$

16. $4a + 2b - 6c = 2$
$6a + 3b - 9c = 3$
$8a + 4b - 12c = 6$

17. $2r + s + t = 14$
$-r - 3s + 2t = -2$
$4r - 6s + 3t = -5$

CONTENTS

18. $3x + y + z = 4$
$2x + 2y + 3z = 3$
$x + 3y + 2z = 5$

19. $4a - 2b + 8c = 30$
$a + 2b - 7c = -12$
$2a - b + 4c = 15$

20. $2r + s + t = 7$
$r + 2s + t = 8$
$r + s + 2t = 11$

21. $6x + 2y + 4z = 2$
$3x + 4y - 8z = -3$
$-3x - 6y + 12z = 5$

22. $r + s + t = 5$
$2r - 7s - 3t = 13$
$\frac{1}{2}r - \frac{1}{3}s + \frac{2}{3}t = -1$

23. $2a - b + 3c = -7$
$4a + 5b + c = 29$
$a - \frac{2b}{3} + \frac{c}{4} = -10$

24. The sum of three numbers is 20. The second number is 4 times the first, and the sum of the first and third is 8. Find the numbers.

25. The sum of three numbers is 12. The first number is twice the sum of the second and third. The third number is 5 less than the first. Find the numbers.

26. **TRAVEL** Jonathan and members of his Spanish Club are going to Costa Rica over spring break. Before his trip, he purchases 10 travelers checks in denominations of $20, $50, and $100, totaling $370. He has twice as many $20 checks as $50 checks. How many of each type of denomination of travelers checks does he have?

DINING For Exercises 27 and 28, use the following information.
Maka loves the lunch combinations at Rosita's Mexican Restaurant. Today however, she wants a different combination than the ones listed on the menu.

27. Assume that the price of a combo meal is the same price as purchasing each item separately. Find the price for an enchilada, a taco, and a burrito.

28. If Maka wants 2 burritos and 1 enchilada, how much should she plan to spend?

Lunch Combo Meals

1. Two Tacos, One Burrito $6.55

2. One Enchilada, One Taco, One Burrito..............................$7.10

**3.** Two Enchiladas, Two Tacos................................$8.90

BASKETBALL For Exercises 29 and 30, use the following information.
In the 2000–2001 season, Minnesota's Katie Smith was ranked first in the WNBA for total points and three-point goals made. She scored 646 points making 355 shots, including 3-point field goals, 2-point field goals, and 1-point free throws. She made 27 more 2-point field goals than 3-point field goals.

29. Write a system of three equations that represents the number of goals Katie Smith made.

30. Find the number of each type of goal she made.

Online Research **Data Update** What are the current rankings for the WNBA? Visit www.algebra2.com/data_update to learn more.

31. **CRITICAL THINKING** The general form of an equation for a parabola is $y = ax^2 + bx + c$, where (x, y) is a point on the parabola. Determine the values of a, b, c for the parabola at the right. Write the general form of the equation.
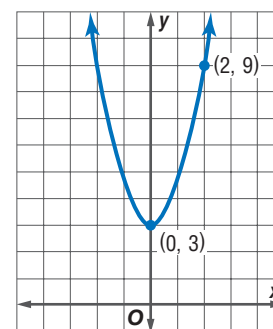
CONTENTS

32. WRITING IN MATH   Answer the question that was posed at the beginning of the lesson.

How can you determine the number and type of medals U.S. Olympians won?

Include the following in your answer:
- a demonstration of how to find the number of each type of medal won by the U.S. Olympians, and
- a description of another situation where you can use a system of three equations in three variables to solve a problem.

33. If $a + b = 16$, $a - c = 4$, and $b - c = -4$, which statements are true?

   I. $b + c = 12$

   II. $a - b = 8$

   III. $a + c = 20$

   (A) I only   (B) II only
   (C) I and II only   (D) I, II, and III

34. If $x + y = 1$, $y + z = 10$, and $x + z = 3$, what is $x + y + z$?
   (A) 7   (B) 8   (C) 13   (D) 14

Maintain Your Skills

Mixed Review   **35. PAPER**   Wood pulp can be converted to either notebook paper or newsprint. The Canyon Pulp and Paper Mill can produce at most 200 units of paper a day. Regular customers require at least 10 units of notebook paper and 80 units of newspaper daily. If the profit on a unit of notebook paper is $500 and the profit on a unit of newsprint is $350, how many units of each type of paper should the mill produce each day to maximize profits?   *(Lesson 3-4)*

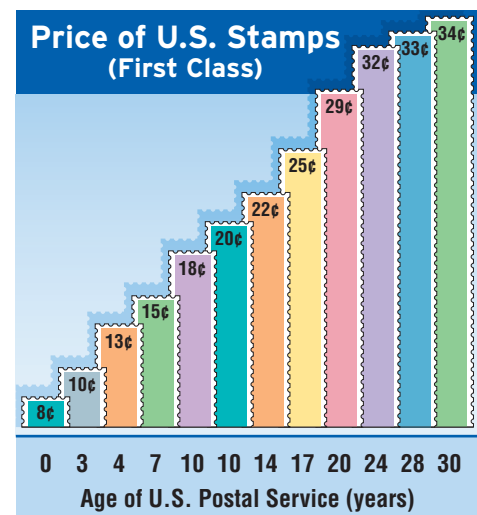**Solve each system of inequalities by graphing.**   *(Lesson 3-3)*

36. $y \leq x + 2$ **37.** $4y - 2x > 4$ **38.** $3x + y \geq 1$
    $y \geq 7 - 2x$ $3x + y > 3$ $2y - x \leq -4$

STAMPS   For Exercises 39 and 40, use the following information.
The table shows the price for first-class stamps since the U.S. Postal Service was created on July 1, 1971.   *(Lesson 2-5)*

39. Write a prediction equation for this relationship.

40. Predict the price for a first-class stamp issued in the year 2010.

Price of U.S. Stamps (First Class)

Age (years)	0	3	4	7	10	10	14	17	20	24	28	30
Price	8¢	10¢	13¢	15¢	18¢	20¢	22¢	25¢	29¢	32¢	33¢	34¢

Age of U.S. Postal Service (years)

Simplify each expression.   *(Lesson 1-2)*

41. $5x + 2y - 4x + y$ **42.** $(4z + 1) - (6z - 7)$

43. $(8s - 5t) + (9t + s)$ **44.** $4(6a + 5b) - 2(3a + 2b)$

CONTENTS

Vocabulary and Concept Check

bounded region (p. 129)
consistent system (p. 111)
constraints (p. 129)
dependent system (p. 111)

elimination method (p. 118)
feasible region (p. 129)
inconsistent system (p. 111)
independent system (p. 111)

linear programming (p. 130)
ordered triple (p. 139)
substitution method (p. 116)
system of equations (p. 110)

system of inequalities (p. 123)
unbounded region (p. 130)
vertices (p. 129)

Choose the letter of the term that best matches each phrase.

1. the inequalities of a linear programming problem
2. a system of equations that has an infinite number of solutions
3. the region of a graph where every constraint is met
4. a method of solving equations in which one equation is solved for one variable in terms of the other variable
5. a system of equations that has at least one solution
6. a method of solving equations in which one variable is eliminated when the two equations are combined
7. the solution of a system of equations in three variables (x, y, z)
8. a method for finding the maximum or minimum value of a function
9. a system of equations that has no solution
10. a region in which no maximum value exists

a. consistent system
b. dependent system
c. constraints
d. inconsistent system
e. elimination method
f. feasible region
g. linear programming
h. ordered triple
i. substitution method
j. unbounded region

Lesson-by-Lesson Review

3-1 *Solving Systems of Equations by Graphing*

See pages 110–115.

Concept Summary

- The solution of a system of equations can be found by graphing the two lines and determining if they intersect and at what point they intersect.
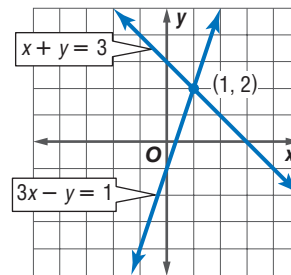
Example   **Solve the system of equations by graphing.**

$x + y = 3$
$3x - y = 1$

Graph both equations on the same coordinate plane.

The solution of the system is $(1, 2)$.

Exercises   **Solve each system of equations by graphing.**   *See Example 1 on page 110.*

11. $3x + 2y = 12$
    $x - 2y = 4$

12. $8x - 10y = 7$
    $4x - 5y = 7$

13. $y - 2x = 8$
    $y = \frac{1}{2}x - 4$

14. $20y + 13x = 10$
    $0.65x + y = 0.5$

CONTENTS

3-2 *Solving Systems of Equations Algebraically*

Concept Summary

- In the substitution method, one equation is solved for a variable and substituted to find the value of another variable.
- In the elimination method, one variable is eliminated by adding or subtracting the equations.

Examples

1 Use substitution to solve the system of equations.

$x = 4y + 7$
$y = -3 - x$

Substitute $-3 - x$ for y in the first equation.

$x = 4y + 7$	First equation
$x = 4(-3 - x) + 7$	Substitute $-3 - x$ for y.
$x = -12 - 4x + 7$	Distributive Property
$5x = -5$	Add $4x$ to each side.
$x = -1$	Divide each side by 5.

Now substitute the value for x in either original equation.

$y = -3 - x$	Second equation
$y = -3 - (-1)$ or -2	The solution is $(-1, -2)$.

2 Use the elimination method to solve the system of equations.

$3x - 2y = 8$
$-x + y = 9$

Multiply the second equation by 2. Then add the equations to eliminate the y variable.

$3x - 2y = 8$ **Multiply by 2.** $3x - 2y = 8$
$-x + y = 9$ $(+) -2x + 2y = 18$
                                                   $ x = 26$ Add the equations.

Replace x with 26 and solve for y.

$3x - 2y = 8$	Original equation.
$3(26) - 2y = 8$	Replace x with 26.
$78 - 2y = 8$	Multiply.
$-2y = -70$	Subtract 78 from each side.
$y = 35$	The solution is $(26, 35)$.

Exercises   Solve each system of equations by using either substitution or elimination.   *See Examples 1–4 on pages 116–119.*

15. $x + y = 5$
$ 2x - y = 4$

16. $2x - 3y = 9$
$ 4x + 2y = -22$

17. $7y - 2x = 10$
$ -3y + x = -3$

18. $-2x - 6y = 0$
$ 3x + 11y = 4$

19. $3x - 5y = -13$
$ 4x + 2y = 0$

20. $x + y = 4$
$ x - y = 8.5$

CONTENTS

3-3 Solving Systems of Inequalities by Graphing

See pages 123–127.

Concept Summary

- A solution of a system of inequalities is found by graphing the inequalities and determining the intersection of the graphs.
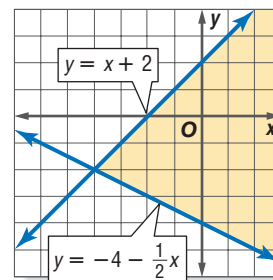
Example Solve the system of inequalities by graphing.

$y \leq x + 2$

$y \geq -4 - \dfrac{1}{2}x$

Graph each inequality and shade the intersection.

Exercises Solve each system of inequalities by graphing.
See Examples 1–3 on pages 123–124.

21. $y \leq 4$
    $y > -3$

22. $|y| > 3$
    $x \leq 1$

23. $y < x + 1$
    $x > 5$

24. $y \leq x + 4$
    $2y \geq x - 3$

3-4 Linear Programming

See pages 129–135.

Concept Summary

- The maximum and minimum values of a function are determined by linear programming techniques.

Example The available parking area of a parking lot is 600 square meters. A car requires 6 square meters of space, and a bus requires 30 square meters of space. The attendant can handle no more than 60 vehicles. If a car is charged $3 to park and a bus is charged $8, how many of each should the attendant accept to maximize income?

Let c = the number of cars and b = the number of buses.

$c \geq 0$, $b \geq 0$, $6c + 30b \leq 600$, and $c + b \leq 60$

Graph the inequalities. The vertices of the feasible region are (0, 0), (0, 20), (50, 10), and (60, 0).

The profit function is $f(c, b) = 3c + 8b$. The maximum value of $230 occurs at (50, 10). So the attendant should accept 50 cars and 10 buses.
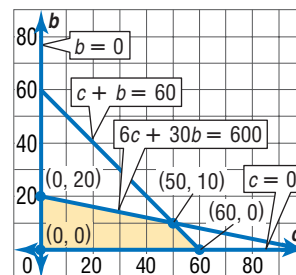
Exercise *See Example 3 on page 131.*

25. **MANUFACTURING** A toy manufacturer is introducing two new dolls, My First Baby and My Real Baby. In one hour, the company can produce 8 First Babies or 20 Real Babies. Because of demand, the company produces at least twice as many First Babies as Real Babies. The company spends no more than 48 hours per week making these two dolls. The profit on each First Baby is $3.00, and the profit on each Real Baby is $7.50. Find the number and type of dolls that should be produced to maximize profit.

CONTENTS

Chapter

3 For More ...

- Extra Practice, see pages 832–833.
- Mixed Problem Solving, see page 864.

3-5 Solving Systems of Equations in Three Variables

Summary

- A system of three equations in three variables can be solved algebraically by using the substitution method or the elimination method.

Example Solve the system of equations.

$x + 3y + 2z = 1$
$2x + y - z = 2$
$x + y + z = 2$

Step 1 Use elimination to make a system of two equations in two variables.

$x + 3y + 2z = 1$ Multiply by 2. $2x + 6y + 4z = 2$ First equation
$2x + y - z = 2$ $(-)\ 2x + y - z = 2$ Second equation
                                       $\overline{\qquad\qquad 5y + 5z = 0}$ Subtract.

$\quad x + 3y + 2z = 1$ First equation
$(-)\ x + y + z = 2$ Third equation
$\overline{\qquad 2y + z = -1}$ Subtract to eliminate x.

Step 2 Solve the system of two equations.

$5y + 5z = 0$ Multiply by 5. $5y + 5z = 0$
$2y + z = -1$ $(-)\ 10y + 5z = -5$
                                 $\overline{\qquad -5y \qquad = 5}$ Subtract to eliminate z.
                                 $\qquad\quad y = -1$ Divide by -5.

Substitute -1 for y in one of the equations with two variables and solve for z.

$5y + 5z = 0$ Equation with two variables
$5(-1) + 5z = 0$ Replace y with -1.
$5z = 5$ Add 5 to each side.
$z = 1$ Divide each side by 5.

Step 3 Substitute -1 for y and 1 for z in one of the equations with three variables.

$2x + y - z = 2$ Original equation with three variables
$2x + (-1) - 1 = 2$ Replace y with -1 and z with 1.
$2x = 4$ Add 2 to each side.
$x = 2$ Divide each side by 2.

The solution is $(2, -1, 1)$.

Exercises Solve each system of equations. *See Examples 2–4 on pages 140–141.*

26. $x + 4y - z = 6$
$\quad\ \ 3x + 2y + 3z = 16$
$\quad\ \ 2x - y + z = 3$

27. $2a + b - c = 5$
$\quad\ \ a - b + 3c = 9$
$\quad\ \ 3a - 6c = 6$

28. $e + f = 4$
$\quad\ \ 2d + 4e - f = -3$
$\quad\ \ 3e = -3$

CONTENTS

Vocabulary and Concepts

Choose the word or term that best completes each statement or phrase.

1. Finding the maximum and minimum value of a linear function subject to constraints is called *(linear, polygonal)* programming.

2. The process of adding or subtracting equations to remove a variable and simplify solving the system of equations is called *(substitution, elimination)*.

3. If a system of three equations in three variables has one solution, the graphs of the equations intersect in a *(point, plane)*.

Skills and Applications

Solve each system of equations by graphing, substitution, or elimination.

4. $-4x + y = -5$
   $2x + y = 7$

5. $x + y = -8$
   $-3x + 2y = 9$

6. $3x + 2y = 18$
   $y = 6x - 6$

7. $-6x + 3y = 33$
   $-4x + y = 16$

8. $-7x + 6y = 42$
   $3x + 4y = 28$

9. $2y = 5x - 1$
   $x + y = -1$

Solve each system of inequalities by graphing.

10. $y \geq x - 3$
    $y \geq -x + 1$

11. $x + 2y \geq 7$
    $3x - 4y < 12$

12. $3x + y < -5$
    $2x - 4y \geq 6$

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and the minimum values of the given function.

13. $5 \geq y \geq -3$
    $4x + y \leq 5$
    $-2x + y \leq 5$
    $f(x, y) = 4x - 3y$

14. $x \geq -10$
    $1 \geq y \geq -6$
    $3x + 4y \leq -8$
    $2y \geq x - 10$
    $f(x, y) = 2x + y$

MANUFACTURING For Exercises 15 and 16, use the following information.
A sporting goods manufacturer makes a $5 profit on soccer balls and a $4 profit on volleyballs. Cutting requires 2 hours to make 75 soccer balls and 3 hours to make 60 volleyballs. Sewing needs 3 hours to make 75 soccer balls and 2 hours to make 60 volleyballs. Cutting has 500 hours available, and Sewing has 450 hours available.

15. How many soccer balls and volleyballs should be made to maximize the profit?

16. What is the maximum profit the company can make from these two products?

Solve each system of equations.

17. $x + y + z = -1$
    $2x + 4y + z = 1$
    $x + 2y - 3z = -3$

18. $x + z = 7$
    $2y - z = -3$
    $-x - 3y + 2z = 11$

19. **SHOPPING** Carla bought 3 shirts, 4 pairs of pants, and 2 pairs of shoes for a total of $149.79. Beth bought 5 shirts, 3 pairs of pants, and 3 pairs of shoes totaling $183.19. Kayla bought 6 shirts, 5 pairs of pants, and a pair of shoes for $181.14. Assume that all of the shirts were the same price, all of the pants were the same price, and all of the shoes were the same price. What was the price of each item?

20. **STANDARDIZED TEST PRACTICE** Find the point at which the graphs of $2x + 3y = 7$ and $3x - 4y = 2$ intersect.
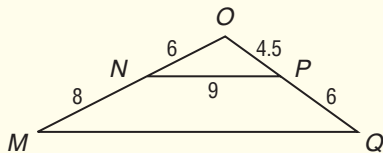
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Chapter 3 — Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. What is the slope of any line parallel to the graph of $6x + 5y = 9$?

- (A) -6
- (B) $-\dfrac{6}{5}$
- (C) $\dfrac{2}{3}$
- (D) 6

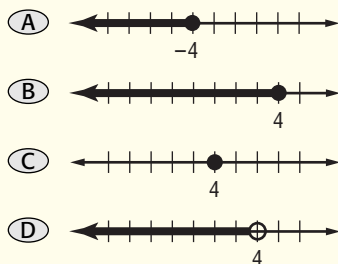**2.** In the figure, $\triangle MOQ$ is similar to $\triangle NOP$. What is the length of $\overline{MQ}$?

- (A) 12
- (B) 12.5
- (C) 19
- (D) 21

3. If $3x - y = -3$ and $x + 5y = 15$, what is the value of y?
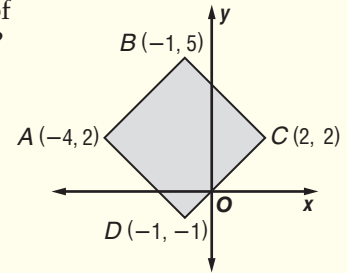
- (A) -3
- (B) 0
- (C) 1
- (D) 3

4. When 3 times x is increased by 4, the result is less than 16. Which of the following is a graph of the real numbers x that satisfy this relationship?

5. What is the area of the square $ABCD$?

- (A) 27 units^2
- (B) $9\sqrt{2}$ units^2
- (C) 18 units^2
- (D) $12\sqrt{2}$ units^2

6. Twenty-seven white cubes of the same size are put together to form a larger cube. The larger cube is painted red. How many of the smaller cubes have exactly one red face?

- (A) 4
- (B) 6
- (C) 9
- (D) 12

7. Find the value of $|-4| \cdot |3|$.

- (A) -12
- (B) -1
- (C) 7
- (D) 12

8. If two sides of a triangle measure 30 and 60, which of the following *cannot* be the measure of the third side?

- (A) 30
- (B) 31
- (C) 40
- (D) 60

9. Marcus tried to compute the average of his 8 test scores. He mistakenly divided the correct total S of his scores by 7. The result was 12 more than what it should have been. Which equation would determine the value of S?

- (A) $8S - 12 = 7S$
- (B) $\dfrac{S}{7} = \dfrac{S + 12}{18}$
- (C) $\dfrac{S}{7} + 12 = \dfrac{S}{8}$
- (D) $\dfrac{S}{7} - 12 = \dfrac{S}{8}$

10. If $x = -2$, then $15 - 3(x + 1) =$

- (A) $6.$
- (B) $12.$
- (C) $18.$
- (D) $21.$

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Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

11. Six of the 13 members of a club are boys, and the rest are girls. What is the ratio of girls to boys in the club?

12. The integer k is greater than 50 and less than 100. When k is divided by 3, the remainder is 1. When k is divided by 8, the remainder is 2. What is one possible value of k?

13. The area of the base of the rectangular box shown at the right is 35 square units. The area of one of the faces is 56 square units. Each of the dimensions a, b, and c is an integer greater than 1. What is the volume of the rectangular box?
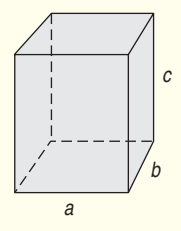
14. Four lines on a plane intersect in one point, forming 8 equal angles that are nonoverlapping. What is the measure, in degrees, of one of these angles?

15. What is the greatest of five consecutive integers if the sum of these integers equals 135?

16. If the perimeter of a rectangle is 12 times the width of the rectangle, then the length of the rectangle is how many times the width?

17. Points A, B, C, and D lie in consecutive order on a line. If $AC = \frac{4}{3}AB$ and $BD = 6BC$, then what is $\frac{AB}{CD}$?

18. The average (arithmetic mean) of the test scores of a class of x students is 74, and the average of the test scores of a class of y students is 88. When the scores of both classes are combined, the average is 76. What is the value of $\frac{x}{y}$?

Part 3 Quantitative Comparison

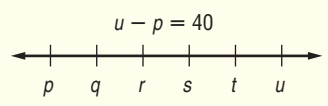**Compare the quantity in Column A and the quantity in Column B. Then determine whether:**

(A) **the quantity in Column A is greater,**

(B) **the quantity in Column B is greater,**

(C) **the two quantities are equal, or**

(D) **the relationship cannot be determined from the information given.**

Column A	Column B
19. the percent increase from 75 to 100	the percent decrease from 100 to 75

20. $3x + 12 = 5x - 3$

$2x$	9

21.

the sum of two different prime numbers if each number is less than 8	the sum of two different positive even integers if each integer is less than 8

22. Integers p, q, r, s, t, and u are equally spaced on the number line.

$$u - p = 40$$

$t - s$	8

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